UNIVERSITY OF CAPE COAST

CHARACTERISATION OF VOLATILITY OF RETURNS IN EQUITY MARKETS OF SUB-SAHARAN AFRICA

BY

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DECLARATION

Candidate's Declaration

I hereby declare that this thesis is the result of my own original research and that no part of it has been presented for another degree in this university or elsewhere.

Candidate's Signature .... Date 02-07-2021

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Supervisors' Declaration

We hereby declare that the preparation and presentation of the thesis were supervised in accordance with the guidelines on supervision of the thesis laid down by the University of Cape Coast.

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ABSTRACT

Sub-Saharan Africa equity markets have been characterised in various practitioner literature as risky. Unfortunately, not much has been done in extant academic works to properly provide evidence and support to guide investor decision-making in the sub-region. Based on this argument, specifications of various regime switching GARCH models were made with various tail innovations to study the volatility of the returns of the Ghana, Kenya, Nigeria and Botswana exchanges using the daily broad market closing indices. These regime switching models were compared with the single non-switching GARCH models using the Deviance Information Criteria (DIC) for model fit. The study established the dominance of the low volatility regime for Ghana, Kenya and Nigeria with the high volalitility periods interspersed for brief times during the sample period. The opposite was found for the Botswana exchange. The study also established the most and least volatile months of the exchanges by a process of resampling the daily into monthly data and finding the average monthly volatility ranking for the various exchanges. Findings of this study will inform investors in the sub-region equity markets on the behaviour of risk and how they can strategise their trading activities to either take advantage of the volatility or avoid losses. For policymakers, it alerts them of the presence of regimes in the markets and a reminder of when regulation and/or policy promote equity market activity in their respective countries.

KEY WORDS

GARCH

Markov Chain Monte Carlo

Metropolis-Hastings

Price filters

Regime switching

Sub-Saharan equities

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DEDICATION

To Regina, Natty and Shika for the love and care.

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LIST OF ACRONYMS

ADF	Augmented Dickey-Fuller
AIMS	Alternative Investment Market Segment
ASEA	African Securities Exchanges Association
BSI	Botswana Stock Exchange
BSM	Botswana Share Markets
BTCL	Botswana Telecommunication Corporation Limited
CAGE	Cultural, Administrative, Geographic and Economic
DQ	Dynamic Quantile
DGP	Data Generation Process
DIC	Deviance Information Criteria
EAC	East African Community
EGARCH	Exponential GARCH
ETF	Exchange-Traded Fund
FTSE	Financial Times Stock Exchange
GARCH	Generalised Autoregressive Conditional Heteroscedasticity
GAX	Ghana Alternative Market
GED	Generalized Exponential Distribution
GEMS	Growth Enterprise Market Segment
GFIM	Ghana Fixed Income Markets
GJR-GARCH	Glosten-Jagannathan-Runkle GARCH
GoG	Government of Ghana
GSE	Ghana Stock Exchange
IFC	International Finance Corporation
IMF	International Monetary Fund

IOSCO	International Organisation of Security Commissions
JSE	Johannesburg Stock Exchange
KSE	Nairobi Stock Exchange
MCMC	Markov Chain Monte Carlo
MIMS	Main Investment Market Segment
MSCI	Morgan Stanley Composite Index
NASI	NSE All-Share Index
NSE	Nigeria Stock Exchange
PP	Phillips-Perron
QQ	Quantile-Quantile
RNE	Relative Numerical Efficiency
SAP	Structural Adjustment Programme
SSE	Sustainable Stock Exchanges
USD	United State Dollars
VaR	Value-at-Risk
WFE	World Federation of Exchanges
ZW	Zivot-Andrews

CHAPTER ONE

INTRODUCTION

Sub-Saharan Africa has a lot of investment potential but investors have not been able to firmly characterise the risks in the markets. Much of what is known about the equity markets is in the practitioner literature and this tends to be in-house for management and the investor community of these investment houses. There are few extant studies in the academic literature that sought to characterise the risk of investing in the sub-Saharan African markets. Unfortunately, these studies, to start with, are disparate and furthermore based on assumptions that ignore the behaviour of the data coming out of the markets. The economies of sub-Saharan African countries are buffeted by disturbances which are economic, social and political. These influence trade on the various stock exchanges and therefore the pattern of returns of financial assets. Any model that seeks to estimate risk must take these circumstances into account else they will not be measuring the real volatility behaviour in the market and serve no purpose.

This chapter poses the question of whether incorporating regime changes in the risk estimation models are the appropriate approach to use in the sub-Saharan African markets. This is important for both investors and policymakers in the sub-region. Investors in equities in the sub-Saharan markets are well positioned to adapt their trading strategies to profit from the particular regime in place and also manage their risks. For policymakers, knowing the regime shifts in the markets will be a pivotal moment in adjusting the monetary and fiscal policy framework to smoothen the business cycle of the respective countries.

Background to the Study

Volatility is an important metric in investment decision-making. Investors, and in general, finance and economic practitioners, pay particular attention to the movements or fluctuations in the trading of assets in the financial markets. The accurate characterisation of volatility of returns is essential for the pricing of financial assets, risk management and taking decisions about where and what assets to include in their portfolios. It is the quintessential that directs investors on how to optimise their resources to achieve maximum returns whiles minimising the downside risk. As a statistical artefact, however, it is not directly observable in the market. It has to be estimated from historical data or backed out from observed option prices in the markets.

The financial returns for which we seek to estimate the volatility are driven by a variety of factors in the market environment. This assertion was made by Hamilton (1990) and reaffirmed by Hamilton and Susmel (1994) in their study of the performance of different volatility models in the US financial markets. Several authors have followed up since, trying to assign reasons for this phenomenon in the developed and emerging markets and among different asset classes too.

In the developed markets, Ang and Timmermann (2012) and Hardy (2001) carried out studies on the financial markets in general and equity markets respectively and provided evidence of regime switching behaviour in the volatility of returns. Regime switching occurs in time series when the evolving data generation process shows a sudden or abrupt change in behaviour by means of assuming a higher or lower value from its previously typical values. It is a phenomenon that is seen very often in financial markets where prices of assets could surge or fall with consequent decrease or increase in volatility respectively. As the regimes of the market changes, we observe a change in the statistical properties of the data as well.

Regime switching and structural change come under change point problems in financial and economic time series (Kleiber, 2018; Piger, 2009; Guidolin, 2011). Barry and Hartigan (1992) and Carlin, Gelfand and Smith (1992) addressed extensively the theoretical basis of these change point problems. Basically, both problems refer to changes in the non-stationarity nature of the mean, the variance, the autocorrelation or any two or more of these characteristics of the distribution of the time series. It is to be noted that some authors use this term synonymously with structural change, a term that is reserved for the permanent switching of the parameters or properties of the data generation process (DGP). For example, Babikir, Gupta, Mwabutwa and Owusu-Sekyere (2012) studied the Johannesburg Stock Exchange (JSE) employing regime switching generalised autoregressive conditional heteroscedasticity (GARCH) models in estimating volatility but used structural changes to describe the phenomenon. Carrasco (2002) also used the terms synonymously in studying the misspecification in macroeconomic models. In their work on house prices, Than-Thi, Dong and Chen (2019) equally referred to regime-switching as structural breaks when house prices were in effect mean-reverting. However, Piger (2009) addressed the distinction between structural change and regime switching by noting that the former involves a permanent migration of the parameters to a new level whereas the latter is characterised by mean-reverting processes. More recently, Billio, Casarin and Osuntuyi (2016) distinguished between these two terms in their work on Markov regime-switching using Gibbs sampling.

For financial markets, Ang and Bekaert (2004) attributed the changes in regimes to the business cycles that are part of the natural development cycle of the economy. In the emerging markets, Assoe (1998), for example, studied market returns and found evidence of switching in volatility of equities. The author attributed this mainly to the uncertainties that come with the changing regulations usually associated with relatively young markets trying to find their feet. Government experimentation with economic policies, a characteristic of developing economies, also directs the attractiveness of various assets classes in the capital markets and this has huge implications for the volatility of equities (Muralidharan & Niehaus, 2017; Rydqvist, Spizman, & Strebulaev, 2014; Mosley & Singer, 2008).

For the frontier markets, especially those within sub-Saharan Africa, very little has been done in the research of these markets. Coverage by researchers and independent financial analysts is thin and sparse. Quisenberry and Griffith (2010) from the practitioner community provide some glimpses into the frontier market dynamics. Frontier markets are relatively difficult to define exactly. Samarakoon (2011) looks at frontier markets "as relatively small and illiquid markets, even by emerging market standards and include all the markets that are not classified as emerging markets". Thus, the defining characteristic of frontier markets is their size and a lack of liquidity (Zaremba, 2019). International index providers such as Morgan Stanley and Financial Times list comprehensive economic indicators on which markets are classified as developed, emerging or frontier (MSCI Frontier Markets, 2019; FTSE, 2019).

Increasingly, frontier market assets are becoming important additions to global portfolios. Miles (2005) reassures investors that in spite of the globalisation of financial markets, frontier market financial assets returns have relatively low correlations with developed and emerging market returns. This is further supported in a paper by Chen, Chen and Lee (2014) in their study of the composition of international portfolios after the global financial crisis of 2008-2009. It is instructive to note that despite the move towards globalisation of financial markets, Berger, Pukthuanthong and Yang (2011) found that frontier markets have relatively low level of integration with the rest of the world markets. This means investors can make good pickings with the potential of attractive returns. Unfortunately, for sub-Saharan Africa equity markets, there is less in the finance literature on the correct characterisation of the return generation processes that give rise to the risks observed by practitioners. Investors have had to rely on their experiences and rules of thumb from the developed and emerging markets to navigate the sometimes turbulence in the frontier markets. Admittedly, the market environment in the frontier economies is far from benign this makes accurate investment-related information critical to investment and portfolio management. It is a time-honoured theory in finance that those who are unable to get the dynamics in the market environment right are eventually driven out (Alchian, 1950). Thus, surviving in 'new' markets away from home requires that investors are able to forecast with some degree of accuracy and confidence on the direction of the market.

Unfortunately, estimation of volatility models in frontier markets has hitherto followed the GARCH framework of Engle (1982) and Bollerslev (1986) irrespective of the different environments of the markets. For frontier markets in sub-Saharan Africa, the market environment is far from benign and estimating risks with the aforementioned models is likely to lead to mispricing of assets and also the execution of bad risk management strategies. That said, assessing the correct level of volatility in frontier markets and how it evolves over time is nontrivial. This is particularly challenging due to a dearth of market information. A multiplicity of risk factors drive market returns in frontier markets conferring it with characteristics different from that of the emerging markets (Bekaert, Erb, Harvey, & Viskanta, 1998). These factors can be linked to the developments in the underlying economy of the frontier markets.

Expectations regarding the performance of the economy in terms of government or central bank policy with respect to spending and taxation or money supply and interest rates have direct effects differentially on fixed income and equities. Economic policies in most frontier markets which rely very much on the exports of primary commodities is largely experimental. Political upheavals in the form of civil unrests, coups, revolutions, etc. in a country normally scare investors away as they reassess country risks and the probabilities of making losses on their investments. Political risk, according to Busse and Hefeker (2007), increases the cost of doing business and therefore the expected returns demanded by investors in conflict-prone markets. Dimic, Orlov and Piljak (2015) found that political risks are relatively elevated in frontier markets compared to developed and emerging markets.

Suliman and Mollick (2009) document extensively the debilitating effects of communal conflicts on investments in sub-Saharan Africa, claiming a substantial drop in foreign direct investments and subsequent exit of investors from the fragile markets. Cote d'Ivoire is an example in recent memory. By some accounts (see Klapper, Richmond and Tran (2013) and Klaas (2008) among others), the Ivorian economy slowed down substantially following the capital flight that occurred during the decade-long civil war. Foreign direct investments were in negative territory and economic growth remained at one percentage point during the civil war. Such developments in the underlying economy endow the data coming from the markets with nonlinear characteristics best described by piecewise functions with their own means, standard deviations and covariance structure and autocorrelations. Indeed, that is the position of Lamoureux and Lastrapes (1990) who argue that the persistence or near unit root observed in volatility models are due to the wrong characterisation of the data. This leads to an over- and under-estimation of risks in periods of tranquility and upsurge in volatility respectively in the markets.

The transmission of such shocks to the capital markets means a more volatile and challenging environment that drives market returns low as investors dump their holdings and make for the exit. Equally, there are periods of tranquility in the markets with rising valuations accompanied by decent returns for investors. How do we characterize the behaviour of volatility given these observations? Clearly, single-regime GARCH models, the workhorse of volatility modeling of financial asset returns will not suffice here. Single-regime models lead to mis-characterisation of the data generating mechanism;

hence the wrong probability distribution of the returns. More often, this leads to loss of information and as a consequence, wrong inferences would be drawn from the data. Rather than assuming a single-modal distribution of the data, regime switching models drill down to uncover the complexity of the underlying distribution and accordingly estimate means, standard deviations and covariances based on the actual structure of the data.

Given the preceding background, one is inclined to think regime switching will characterize the data generation process (DGP) in the returns of financial assets in general and equities in particular in frontier markets. These will lead directly to the use of regime switching GARCH models in the estimation of risk of financial returns. The construction of volatility measures is such as to estimate the parameters of the model piecewise in a given regime. In effect, the use and reliance of regime switching models in the estimation of volatility is motivated by the behaviour of the data as discussed in Danıelsson (2002). Market data is endogenous of the market environment. It goes therefore that any attempt at modelling volatility based on the data should take into account the environment generating the data. This is the approach adopted by this thesis going forward.

This work will fill the information gap in the literature of sub-Saharan African equity markets by explaining the volatility returns based on the view that they are driven by the economic, social and political factors prevailing in a given economy. Investors, traders and policymakers are going to be guided enormously in decision-making bearing in mind the shifts in regimes in the markets.

Statement of the Problem

Investment and portfolio management, pricing of securities and risk management in general require fine-grained estimation of risk. Risk estimation in frontier markets hitherto relies on classical GARCH models without the incorporation of market regimes that will be typical of the market and data characteristics which are quite different from the developed and to a lesser extent the emerging markets. Frontier markets are subject to frequent upheavals and these transmit to the financial markets. Thus, any attempts at volatility estimation must incorporate regime switching in order to correctly determine the risks associated with the assets in the markets.

For equity markets in sub-Saharan Africa, this is even imperative, given problems with experimentation with monetary and fiscal policies, social upheavals, a lack of quality market information for investment and financial reporting issues. This problem is widely recognized in the practitioner community but has not been given the needed attention in the academic literature. Given the increasing prominence of sub-Saharan African assets in global investment portfolios, there arises the need to address this gap in the academic literature and place the markets on the proper pedestal for both institutional and individual investor.

Objectives of the Study

The main objective of this study is to adequately characterise the volatility of equities in selected financial markets in sub-Saharan Africa using quantitative risk models.

The specific objectives are to:

1. Investigate the nature of evidence of regime switching in equity markets in some selected sub-Saharan African markets.
2. Determine the regime switching GARCH model which best depicts the returns in each of the selected markets.
3. Comapare the obtained regime switching models with existing single-regime models.
4. Examine whether regime-switching induces heavy-tailed distributions in the returns for the selected equity markets.

Research Questions

From the foregoing, the questions addressed by this study are:

1. What is the nature of evidence of regime switching in equities in frontier sub-Saharan African markets?
2. Which regime switching GARCH models will characterise the respective returns in these markets?
3. How do regime-switching models compare with single-regime models?
4. Do regime-switching induce heavy-tailed distributions in the returns from the equity markets?

Scope of the Study

The study concentrates on a sample of sub-Saharan African equity markets. The classification of the markets relied on that provided by MSCI (2019) and FTSE (2019). The sub-Saharan African region has been segmented into west, east, central and south with a country each from these regions except for West Africa where Ghana and Nigeria where selected. Nigeria Stock Exchange (NSE) was selected in addition to Ghana Stock Exchange

(GSE) because it is the largest equity market in sub-Saharan Africa. The size of the NSE, number and diversity of listed firms makes it significant for investors. Nairobi Stock Exchange (KSE) and Botswana Stock Exchange (BSI) are selected from east and southern Africa regions respectively. We could not get data on markets in the central African region.

Selection was loosely based on the Cultural, Administrative, Geographic and Economic (CAGE) distance principles espoused by Ghemawat (2001). The CAGE framework is based on the similarity between countries as a result of geographic proximity. By this, a country chosen to represent a geographic region will have similar economic, social and political characteristics of that region as a whole.

We present an overview of the selected stock exchanges as follows:

Ghana Stock Exchange

The GSE was established in July, 1989 under Ghana Companies Code, 1973. It started operations in November, 1990, with initial trading through a call-over system with eleven equities, three brokers and a market capitalisation in current terms of USD0.66 million. The GSE has followed a trajectory of development as any other bourse migrating to a manual continuous auction trading system to now an automated auction market made up of three market segments with forty-one equities, seventy-six Government of Ghana (GoG) bonds and notes, corporate instruments, an exchange-traded fund (ETF) and a thriving twenty-two brokerages. It has a market capitalisation of USD13.3 billion as of December, 2018. According to Hesse (2019), the GSE comes under the regulatory supervision of the Bank of Ghana, the Securities and

Exchange Commission of Ghana, the National Pensions Regulatory Authority, and the Nationa Insurance Commission.

The GSE is constituted of three market segments, namely, Main Equity Market listing the equities and ETFs, the Ghana Fixed Income Markets (GFIM) listing both government and corporate debt instruments, and the Ghana Alternative Market, an initiative to carter for the listing of small-and-medium scale enterprises. Table 1 has the statistics of the recent performance of the exchange.

Table 1: The market statistics of the GSE as of year ending 2018

	2014	2015	2016	2017	2018
Equity market cap (USD billions)	23.83	16.32	13.17	13.68	13.29
Domestic market cap (USD billions)	5.14	3.2	2.72	3.78	5.52
Debt market value (USD billions)	1.14	1.55	5.65	6.68	7.62
Number of listings (Main and GAX)	38	42	44	43	42
New equity securities listed	1	4	2	2	2
Delistings	0	0	0	1	3
Number of issues (GFIM)	-	170	125	129	129
Value traded (Equity USD millions)	128	71	61	121	143
Turnover (Debt USD millions)	-	2860	4192	7140	8232
Equity market returns, GSE -CIa (%)	5.4	-11.77	-15.33	52.73	-0.29
Avg. daily equity volume traded (millions)	0.83	0.99	1.02	1.3	0.81
Market cap/GDP (%)	41.4	31.66	24.5	22.91	20.34

Source: GSE, NPRA and SEC (2019)

In general, we see a decline in the stock market valuation across time compared to the other related economic indicators. The number of issues has increased but there is a corresponding decrease on the stock prices in the market as indicated by the trends in the volumes traded and the equity market capitalisation. The trend in the ratio of market capitalisation to GDP is on the decline. This indicates an increase in economic growth whereas the stock market has not shown corresponding increase. The number of issues has remained fairly stable after an initial drop but these new shares were probably less valued by the markets.

The GSE faces some challenges. At a domestic equity market capitalisation to GDP and total equity market capitalisation to GDP of 8.03% and 19.35%, respectively, Hesse (2019) notes investor flight to the safety of sovereign debt instruments issued by GoG. This investor flight is mainly due to the perceived risks associated with equities, especially at the time of reforms in the financial sector of the economy and the continuing depreciation of the Cedi compared to the returns on equities. It is noteworthy that this flight to safety has the effect of increasing volatility and turnover in the equities and this is registered in the average value traded in the equities until 2018. Furthermore, the trend in market capitalisation to GDP ratio indicated a stagnated market even as GDP continue to grow.

Nairobi Stock Exchange

Kenya has the oldest stock exchange in the sample. The Nairobi Stock Exchange (hereafter KSE to distinguish it from the Nigeria Stock Exchange which has the same abbreviation) started in 1954 as the bourse serving Kenya, Uganda and Tanzania under what was known as the East African Community

(EAC). It became a wholly Kenyan outfit following the collapse of the EAC in 1977 and remained a purely Kenyan entity till today.

Ramji, Wairimu, Mwita and Mwanyasi (2019) reported that the KSE's equities consist of the Main Investment Market Segment (MIMS), the Alternative Investment Market Segment (AIMS), and the Growth Enterprise Market Segment (GEMS). The broad market indices are made up of the NSE All-Share Index (NASI), NSE 20, FTSE NSE Kenya 15, and FTSE NSE Kenya 25. The NASI consists of the sixty-three (63) listed companies and it is the main pulse of the economy of Kenya. In addition, the KSE has a fixed-income and exchange-trade fund together with some derivatives segments. In terms of numbers, the financial services sector with banks included makes about 48% of the exchange. The single market capitalisation is contributed largely by Safaricom PLC with $1 billion in traded equity out of a market capitalisation of $22.60 billion for the KSE as a whole as of July, 2019.

As any other frontier market, the exchange suffers from uncertainty due to the changing regulatory landscape, low market liquity and contagion to US Federal Reserve monetary policy changes due to the number of foreign investors with an eye on US interest rates. Table 2 shows a summary of activity on the KSE for 2008-2017 periods.

Table 2: Summary of activity on the KSE

Year	Equity Turnover (KShs. Bn)	Share Volume (Mn)	End of Period KSE 20-Share Index	End of Period Market Cap (KShs. Bn)
2008	97.52	5856.54	3521.18	853.88
2009	38.16	3160.03	3247.44	834.17
2010	110.32	7545.79	4432.6	1166.99
2011	78.06	5721.83	3205.02	868.24
2012	86.79	5464.23	4133.02	1272
2013	155.75	7665.92	4926.97	1920.72
2014	215.73	8133.67	5112.65	2316
2015	209.38	6812.14	4040.75	2053.52
2016	147.18	5813.49	3186.21	1931.61
2017	171.61	7065.36	3711.94	2521.77

Source: KSE/Capital Markets Authority (2019)

Market turnover in the KSE has fluctuated similar to market capitalisation for the period 2008-2017. The All-Share Index has similarly been choppy. Four continuous years, 2011-2014, saw a consistent increase in both turnover and the index levels.

Nigeria Stock Exchange

The NSE with an equity market capitalisation of $32.13 billion is the largest and most diversified exchange in the sample. It predated the colonial times when the British government used it to issue bonds for construction projects in 1946. On attaining independence, it was formalised into the Lagos Stock Exchange in 1960 and later transitioned into the Nigerian Stock Exchange. Following independence, the policy framework of Nigeria was toward indigenisation of businesses; hence the exchange could not expand through foreign participation to really play its role in mobilising capital for economic expansion. Uduanu (2019) traces the history of the opening up of the bourse to foreign participation starting 1986 when world crude oil prices tumbled and Nigeria was forced to accede to economic liberalisation programme known as Structural Adjustment Programme (SAP) as a precondition for accessing International Monetary Fund (IMF) aid. The change brought by this led to a wave of listing of firms and the exchange continues to grow to this day.

As of the end of 2018, the NSE listed 169 equities, 9 exchange-traded assets, 5 real estate investment trusts and 130 fixed-income securities. As part of its internationalisation efforts, the NSE is a member of the World Federation of Exchanges (WFE), Sustainable Stock Exchanges (SSE), the International Organisation of Security Commissions (IOSCO) and the African Securities Exchanges Association (ASEA). By far, the NSE is the most liquid and deeper of all markets among sub-Saharan Africa. Unfortunately, the NSE is more prone to turbulence whenever markets in the developed countries are gyrating as it happened in 2008 global financial crisis (Adamu, 2009; Offoing, Riman, & Godwin, 2018). This is the result of deep linkages via foreign investor participation in the NSE (Abdullahi, 2017).

Compared with other frontier markets outside of sub-Saharan Africa, however, the NSE remains small with market composition largely concentrated around the financial services (33%), materials (32%) and consumer products (27%). Performance of the NSE has been mainly dictated by changing legislation, especially to do with capital controls occasioned by the crude oil prices on the world market. Table 3 shows market activity on the NSE to the year ending 2018.

Table 3: Performance of the NSE for the period 1985-2018

	1985	1990	1995	2000	2005	2010	2015	2018
Equity market capitalisation (USD million)	5,511	2,470	2,606	4,235	19,562	52,527	50,130	32,133
Value traded (USD million)	0.06	0.03	0.08	1	7.9	21.2	19.9	14.1
Stock market turnover ratio	0.3	0.4	1.1	7.4	12.3	12.3	9.3	9.5
Average price-to-earnings ratio	4.3	7	6.8	7.1	12.8	10	17.8	9.3
Average dividend yield (%)	10.6	12	7.9	7.5	9.5	5.8	5.9	5.6
Number of stocks	96	131	181	195	214	217	190	169

Source: Central Bank of Nigeria, Nigerian Stock Exchange, Nigeria Securities and Exchange Commission, and Bloomberg (2019)

The NSE has experience consistent growth largely except in the latter years where market indicators have remained almost flat. Stock turnover increased consistently from 1985-2010 which indicated market liquidity. There was a dip however in the the intervening period to 2015 perhaps coinciding with the drop in the world market price of crude oil, Nigeria's main export commodity.

Botswana Stock Exchange

The BSI dates 1989 with the formation of the Botswana Share Markets (BSM). The BSM eventually transformed to the BSI through legislative instruments in the latter part of 1994 and it commenced operations with its ownership consisting of brokers via proprietary rights and the government of Botswana in 1995. The bourse currently trades shares, fixed-income instruments and exchange-traded funds denominated in foreign and the local currency known as the Pula.

The listed firms on BSI are engaged in a broad range of business sectors ranging from agriculture, mining, energy, financial services, security services among many; with mining representing the single largest activity of companies holding a share of 85.2% according to Bolokwe and Sedimo (2019). The market is dominated by institutional investors. Retail investors hold about 5% of the market. Table 4 shows the compositions of the listing on the BSI in the period from 2014 to 2018.

Table 4: Composition of the BSI up to year ending 2018

	2014	2015	2016	2017	2018
New listings	1	1	2	3	2
Delisting	1	4	0	2	2
Domestic companies listed	23	22	24	24	26
Foreign companies listed	12	10	10	11	9
Total	35	32	34	35	35

Source: BSI (2019)

BSI is attracting the attention of some regional and international firms as well as larger privatised companies like the Botswana Telecommunication Corporation Limited (BTCL). Like any frontier market, however, the BSI suffers from shallow liquity because institutional investors, for example, the pension funds who dominate the market are long term investors who buy and hold instead of trade assets in the market. Currently, the BSI is making efforts to broaden the financial instruments available in the market with the hope of attracting retail investors.

Significance of the Study

Financial markets follow the developments in their economies and thus go through phases of high and low volatility. Excessive volatility resulting from turbulent markets can be disruptive, throwing investment strategies into chaos as a country scrambles to find the right balance of both monetary and fiscal policies to contain the situation. Financial markets are the pulse of a country's economy. Every significant development in a given country, be it economic, social or political, gets transmitted through financial markets; hence regime switching should correctly capture the changing volatility in returns on financial assets. Therefore, this study characterising the volatility of equities in the financial markets is instructive for the investment decision-making in the frontier equity markets of sub-Saharan Africa.

There are lots of studies on the nature of heteroscedasticity across the selected countries (*See* Atoi (2014), Olowe (2009), Nortey, Asare and Mettle (2015), Adjasi (2009), Esman Nyamongo and Misati (2010), Rao and Moseki (2011), Afuecheta, Utazi, Ranganai and Nnanatu (2020), among others). However, none of these studies have incorporated regime changes into their studies. Furthermore, this study also characterises each individual market with a specific regime switching GARCH model based on its data characteristics. This novelty is the point of departure from the extant literature and throws light on the nature of volatility taking cognisance of the underlying data generation process.

Out of this study, the investor community will correctly assess the risks associated with trading of financial assets in a multi-state heteroscedastic environment, thus being positioned to fine-tune their strategies appropriately. Also, the pricing of financial assets in these markets is improved as a result of correctly estimating the volatility in the regimes.

For policy-makers, it is essential to bear in mind the changing nature of market conditions. In gyrating markets, when asset prices are falling and volatility high, monetary and fiscal policies should be accommodating to bring calm to the markets. Firms in the financial sector are particularly vulnerable to economic phases of high inflation and the attendant high interest rate environment. Loan defaults soar and most borrowers go bust. This affects the liquidity positions and financial health of lenders in the economy. In that case, it is essential that policy makes capital adequacy requirements cyclical and sensitive to market regimes.

Delimitations

Equities of various firms are listed on the respective exchanges across the sub-Saharan Africa sub-region. As the topic suggests, the volatility of these equities is what is under investigation in this study. Given the sheer number of listed equities involved, a decision has been made to use the indices which are the composite representations of the all the list shares. Cheng (2020) and McAleer and Da Veiga (2008) are some of the examples of using stock market indices in the analyses.

Limitations of the Study

This study suffers essentially from two limitations. Geographically, it did not cover all the regions of sub-Saharan Africa. It was difficult getting market data from the central region of the frontier sub-Saharan Africa. This region comprises of Cameroon, Gabon, Central African Republic, Equatorial Guinea, Republic of Congo and Chad. Schiereck, Freytag, Grimm and Bretschneider (2018) describes these equity markets as small and with a total market capitalization of nearly €178 million; hence exclusion will not have any significant effect on the findings of this study.

Secondly, this study limits itself to the regime switching versions of GARCH class of models (Bollerslev, 1986; Nelson, 1991; Glosten, Jagannathan, & Runkle, 1993; Engle, 1982). Being an important but unobserved market metric, there has been a proliferation of volatility estimation models in the finance literature since Engle (1982). Among many others, we have stochastic models (Taylor, 1994b), extreme value models

(Embrechts, Klüppelberg, & Mikosch, 2013), implied volatility methods (Black & Scholes, 1973) and a hybrid of GARCH and extreme value models (McNeil & Frey, 2000).

Finally, the study uses a purely reduced rather than structural model approach to investigating the nature of volatility switching in these markets without going behind the data to see what changes where are at play in the economy leading to the regime switching. Structural models as used by Merton (1974) and Black and Scholes (1973) seek the reasons behind the numbers to provide a deeper exposition of the facts behind the figures.

Organisation of the Study

The remainder of the thesis follows the following structure. Chapter two extends work on Chapter one by reviewing extant literature on regime switching models used for volatility estimation in financial markets. The third chapter deals with the methodology and theoretical basis of regime switching GARCH models within the context of Bayesian inference. In Chapter four, we carry out model fitting using data from selected exchanges in sub-Saharan African markets. This is extended to include model adequacy and selection criteria that leads to back-fitting for model fit. Also, the chapter explores the issues on heavy-tailed distribution of the tails and how this relates to the risk measures of value-at-risk. Chapter five concludes the thesis with observations on the chosen models and their implications for portfolio and investment management in the frontier markets of sub-Saharan Africa.

Chapter Summary

In this chapter, the nature of risk as it relates to the the volatility of market returns was introduced. The exchanges of four sub-Saharan African countries have been selected in this study. The selection is based on the CAGE distance theory. The case for the evolution of volatility across these sub-Saharan African markets is made by looking at the individual characteristics of the various exchanges in the most recent years. We enumerated the research questions and looked at the significance including the limitations of the study for both practitioners and policymakers. In the next chapter, a review of the relevant literature on volatility will be undertaken.

CHAPTER TWO

LITERATURE REVIEW

Introduction

This chapter reviews the literature on the behaviour of volatility across the developed, emerging and frontier markets. It looks at the performance of non-switching versus regime switching volatility models on the major classes of financial assets – equities, bonds, commodities – and how risk in these markets is influenced by the returns that are time-based properties of particular markets. As well, the chapter also reviews literature on the role of tail distributions in heightening risk in the market environment through heavy-tails.

The Nature of Market Volatility

Variations in asset returns contain information vital to any investment strategy. Prolonged periods of turbulent markets signal problems in the economy. This can be the result of unfavourable monetary or fiscal conditions, political uncertainties, social upheavals and the like in the economy. On the other hand, orderly markets with low volatility in asset returns signal an optimally operating economy with few upheavals. Each of these phases may be indicative of when the investors may have to lean against the wind or reorder their strategies to take advantage of a given volatility regime depending on their risk preferences. Investor attitudes to risk is summed up in Engle (2004) that having an idea of the risks in the market enables a reordering of attitudes to it. This is at the heart of investment and portfolio management and, by and large, risks management activities in finance.

Risk, modelled as a time-variation metric in asset returns from their mean was first proposed in Engle's (1982) paper and later generalised by Bollerslev (1986) in which the volatility taking on a heteroscedatic model was first considered. Hitherto, asset returns volatility was simply taken as the standard deviation calculated on the historical returns (Ederington & Guan, 2006). This was oblivious of the the day-to-day, season-to-season variations actually observed in the markets during trading. However, it served investors of the day very well. Markets were relatively simple; the products were largely linear and market regulations at the time were easy to understand. Market upheavals were far and in-between.

Starting 1987 when the US stock market crushed in October sending shock waves to global financial markets, the world has experienced increased volatility in returns across various markets – developed, emerging and frontier, and in all types of asset classes (Furbush, 1989; Vandewalle, Boveroux, Minguet, & Ausloos, 1998). Markets have assumed such complexity never experienced in history with financial products such as derivative and options traded with so much leverage and with it, complex market regulations to guide trading. These developments gave rise to ever more complex risk models meant to provide guidance to market agents on how to tame risk. Among many models, the exponential Generalised Autoregressive Conditional Heteroscedasticity of Nelson (1991) and the Glosten-Jagannathan-Runkle-GARCH (GJR-GARCH) proposed by Glosten, Jagannathan and Runkle (Glosten, Jagannathan, & Runkle, 1993) capturing market leverage and asymmetry in the evolution of volatility respectively, became dominant in the financial and economic disciplines. Over time as markets swung to extremes with increasing frequency, additional models such as extreme value theorem volatility methods were developed to accommodate those extremal outcomes (McNeil & Frey, 2000; Embrechts, Klüppelberg, & Mikosch, 2013).

The stochastic volatility (SV) models proposed by Taylor (1982a) and have been used in Kim, Shephard and Chib (Kim, Shephard, & Chib, 1998) and currently used extensively in high-frequency financial returns (Barndorff-Nielsen & Shephard, 2006; Barndorff-Nielsen & Shephard, 2004) looks at volatility as sort of a latent stochastic process in discrete time. But the complexity of SV models coupled with the difficulty in interpreting its estimates limited their use in mainsteam financial modelling (Aı & Kimmel, 2007).

As market became prone to increased swings at the turn of the century, one characteristic observed about asset returns distributions is the increasing presence of fat-tails and pronounced asymmetry. To capture the combined fat-tails and asymmetry of the tails, Harvey (2013) and Creal, Koopman and Lucas (2013) put forward the Dynamic Conditional Score (DCS) models where volatility is driven by a score. Gao and Zhou (2016) and Bernardi and Catania (2016) provide evidence that these class of models do capture very closely the stylised facts of heavy tails and the asymmetry of the distributions.

There is however a growing concern of the failure of these classical GARCH models to capture risks in the face of the increased turbulence in the markets. This fact is acknowledged extensively in the literature. In the equity markets, Bonilla and Sepulveda (2011) examine stocks in the emerging markets and conclude that the GARCH models find it difficult to deal with the data coming out of such markets. Failure of the classical GARCH models was indeed pointed out in an earlier work by McMillan and Speight (2004). In that paper, it was clear the data exhibited characteristics that the model could not capture.

Market data, as much as is time-varying, exhibits epochs in its evolution depending on the changing nature of the underlying causes driving the DGP (Pincus & Kalman, 2004). Ang and Timmerman (2012) identified the underlying changes as mainly due to variations in policies in the monetary and fiscal spaces, market regulation and other secular changes. When markets are orderly, asset returns are stable for the most part. But there comes periods when volatility spikes, albeit for brief periods. Quandt (1958) and Goldfeld and Quandt (1973) formalised this observation into a linear regression specifying two regimes. The authors identified discontinuities in the data series used to estimate the regression equations and reasoned that such switchings gave rise to regimes with distinct means, variances and correlations specific to the data periods. Thus, the data generation processes in economics and finance are prone to changes which should be subject to proper analysis in order to appropriately characterise the defining statistical moments regime-by-regime.

Unfortunately, Quandt's model had two regimes specified *a priori.* In retrospect, this was a relatively simple but important model that captured the reality of the discontinuities in the DGP giving rise to regimes. In some instances, this discontinuity leads to a complete structural break from one state to another. These insights provided the basis for the work of Hamilton (1990) who formalised the theory behind regime changes in time series literature. Hamilton view market data as a function of the underlying dynamics in the giving rise to it. As there are inevitably periods of rising and falling activity directly related to increasing gross domestic growth and recessions respectively, we should expect the data to follow that behaviour. This idea, as simple as it is, was revolutionary in volatility modelling. The modelling assumptions from the perspective of Harvey's model align with a nonlinear data generation process.

Structural breaks and regime switching are real observations in time series and for any model to be useful, this fact has to be accommodated in the modelling assumptions. Indeed, some writers, for example, Babikir, Gupta, Mwabutwa and Owusu-Sekyere (2012), used the terms "structural break" and "regime switching" synonymously in their papers. Song (2014) provides the distinguishing difference between structural breaks and regime switching models noting that the latter denotes the situation where past states are recurrent and the former is where a change in the dynamics of the data means a movement to new permanent states in the guiding parameters of the model. In sum, with structural break, none of the previous states is capable of recurring (Maheu & Gordon, 2008; Koop & Potter, 2007).

Time series models rely on stationarity for their specification. Unfortunately, the literature is mixed on the issue of stationarity for regime switching data. For the case when the data are taken as a single regime, tests for stationarity are well established with Dickey-Fuller and Kwiatkowski–Phillips–Schmidt–Shin (KPSS) tests (Carrion-i-Silvestre & Sansó, 2006; Hobijn, Franses, & Ooms, 2004; Sephton, 1995). Using such tests for regime switching data, however, is known in the literature to bias the results with power gains or losses which are spurious (Busetti & Taylor, 2003). There are well established methods for testing stationarity in structural models as seen in the works of Bai and Perron (2003a; 2003b; 1998). Zivot and Andrews (2002) provide indirect methods for assessing stationarity by establishing change points in the data. Other methods have been established based on both the frequentist and Bayesian methods in the statistical finance literature (see Erdman & Emerson (2007).

Following this empirical assumption about time series returns, research in financial statistics, econometrics and related disciplines incorporating regime switching in time series has exploded. It is now employed in the prediction of assets returns, estimation of volatility and covariances given that there are regimes in the data. This is seen in the developed (Ang & Timmermann, 2012), emerging (Assoe, 1998) and some frontier markets (Balcilar, Demirer, & Hammoudeh, 2013; Khalifa, Hammoudeh, & Otranto, 2014). It has also found applications in particular assets markets; crude oil markets (Wang, Wu, & Yang, 2016; Vo, 2009), natural gas (Geng, Ji, & Fan, 2016), electricity pricing (Janczura & Weron, 2012; Lindström & Regland, 2012), gold markets (Białkowski, Bohl, Stephan, & Wisniewski, 2015; Sopipan, Sattayatham, & Premanode, 2012; Baur & Glover, 2014), among others. Naturally, these markets are very volatile and market players need to estimate volatility accurately in their hedging and trading activities. Regime switching volatility models provide the fine-grained tools for achieving this important goal.

Classification of Markets

Global trading markets are classified into developed, emerging and frontier. A number of factors are considered in putting a country into one of these baskets. These factors are as varied as the level of economic growth, the liquity of trading securities, how industrialised the country is, the political stability of the countries', among others. MSCI (2019), a global index provider, see developed markets as having free trade with a well-run stock market and generally a very high standard of living. On the other hand, it regards the emerging markets as those at the nascent stages of their development, having some characteristics like low sovereign and private debt, a large consuming citizenry and vast natural and human resource base. MSCI characterise frontier markets as less matured with much illiquidity, a huge demographic advantage and not too developed political systems. There is a fourth, less known category called stand-alone markets. Informationally, Speidell (2009) looks at this classification in terms of information flow in the markets and how this influences liquidity of the trading assets. A market with less information flow on the traded assets is likely to be dominated by noise traders. Perhaps, it is such a characteristic that makes the lower tier markets ripe for global investors searching for yield away from their home bases.

Among the most recognised developed markets are US, Britain, France, Germany, Japan, Netherlands, Ireland and Hong Kong. The classification places Brazil, China, India, Argentina, Thailand and South Africa in the emerging category. In the frontier markets, prominent countries include Kuwait, Saudi Arabia, Nigeria, Jamaica, Kenya and Botswana. It is to be noted that a country's level of development usually proxied on the level of GDP has nothing to do with the classification of markets. This distinction is very important given the confusion investors make, sometimes synonymously regarding countries with high GDP as developed markets and those with low GDP as frontier or in some instances as stand-alone markets.

The Nature of Frontier Economies and Stock Markets

Frontier markets are so-called because they are not yet part of global finance according to the definition provided by the International Finance Corporation (IFC) in 1996. IFC's definition encompassed a hitherto unknown group of twenty-one smaller countries it tracked for investors with its frontier composite index (Speidell & Krohne, 2007). The grouping referred to as "frontier markets" has always been a loose collection of markets with potentially rapid growth in gross domestic product (GDP) and market returns which have low correlations with those of the developed and emerging markets. Marshall, Nguyen and Visaltanachoti (2013) similarly, view frontier markets in terms of their maturity compared with emerging and developed markets counterparts, claiming they lack the quality market attributes like transparency, adequate investor protection, good corporate governance and easy access to information flow on their listed firms.

According to Griffin, Lystra and Quisenberry (2015), FTSE, Morgan Stanley Composite Index (MSCI), Russell and S&P have various criteria for the grouping of the world's markets. FTSE Russell's (2019) methodology for the classification of global financial markets range from accessibility to the market by investors; market quality in terms of regulations, multiplicity of trading products, procedures for custodial and selection in trading; size of the market necessary for member of global benchmarks; predictability and the consistency with which the market conducts its procedures; the endurance of markets changes and stability of market regimes to and the cost of implementing any change(s) that warrants a proposal to promote or demote the market from its present classification. A summary of the various countries and their classifications by the index providers is shown in Table 5.

Table 5: Preview of Frontier Markets according to global index providers

	FTSE	MSCI	Russell	S&P		FTSE	MSCI	Russell	S&P
Argentina	✓	✓	✓	✓	Malta	✓		✓	
Bahrain	✓	✓	✓	✓	Morocco		✓		
Bangladesh	✓	✓	✓	✓	Mauritius	✓	✓	✓	✓
Botswana	✓		✓	✓	Namibia				✓
Bulgaria	✓	✓	✓	✓	Nigeria	✓	✓	✓	✓
Colombia				✓	Oman	✓	✓	✓	✓
Côte d'Ivoire	✓			✓	Pakistan		✓	✓	✓
Croatia	✓	✓	✓	✓	Panama				✓
Cyprus	✓		✓	✓	Papua New Guinea			✓	
Ecuador				✓	Qatar	✓	✓	✓	✓
Estonia	✓	✓	✓	✓	Romania	✓	✓	✓	✓
Gabon			✓		Serbia	✓	✓	✓	
Ghana	✓		✓	✓	Slovakia	✓		✓	✓
Jamaica			✓	✓	Slovenia	✓	✓	✓	✓
Jordan	✓	✓	✓	✓	Sri Lanka	✓	✓	✓	✓
Kazakhstan		✓	✓	✓	Trinidad and Tobago			✓	✓
Kenya	✓	✓	✓	✓	Tunisia	✓	✓	✓	✓
Kuwait		✓	✓	✓	Ukraine		✓	✓	✓
Lebanon		✓		✓	United Arab Emirates		✓		✓
Lithuania	✓	✓	✓	✓	Vietnam	✓	✓	✓	✓
Macedonia	✓				Zambia				✓

Source: Schipke (2015)

The classifications in Table 5 reflect the different methodologies used by these index providers. Botswana and Ghana, for example, are classified by three of the four index providers as frontier markets. On the other hand, Kenya and Nigeria, the bigger markets in the sample, are typical frontier markets classified by all the four index providers. These groups are actually fluid and any classification is a reflection of the dynamics prevailing at any given time.

For frontier markets, Barclays (2018), a UK bank, sets out some of their characteristics as having a rapidly growing middle class with a rising economic growth, typically young populations, countries are endowed with rich and unexplored vast natural resources, have relatively low valuations of the financial assets, their governments pursuing macroeconomic reforms including removing barriers to investor business. Dickson (2013) added that frontier markets have healthy market returns compared to their counterparts in the developed and emerging markets.

These characteristics are the ingredients for economic growth and prosperity that provide future markets for firms doing business or potentially have an eye in frontier markets. In terms of size, Schipke (2015) who provides extensive work in the economies of the frontier markets beyond Asia puts the global equity holding in the region of 1-2 percent.

As a group, according to Karolyi (2015) and Kearney (2012), these economies are known to be potentially politically unstable and with governance rife with corruption and markets lacking in robust regulatory frameworks – a key requirement for the smooth functioning of financial markets and an assurance of protection of investments and enforcement of contracts. The last point is underpinned by Odell and Ali (2016) who argue that the presence of legal and regulatory structures is particularly important to investors venturing into unknown markets such as in the frontier economies. Dimic, Orlov and Piljak (2015) studied the impact of political stability on the suitability of a country as a destination for investments across developed, emerging and frontier countries and reached the conclusion that instability in a country's governance has debilitating effects on its financial markets. The other concerns, especially related to financial assets, is their illiquidity. The effect of illiquidity of financial assets and how this increases risk is well documented in literature. Marshall, Nguyen and Visaltanachoti (2013) noted that illiquidity in frontier markets is mainly the result of thin and asynchronous trading on these exchanges. These twin dangers can lead to rapid deterioration of market conditions and eventually catch investors out on their positions in the markets.

Regime-switching Models in Frontier Markets

Concerns about the economy, resulting mainly from monetary and fiscal policies, are a worrisome prospect for investors. In the frontier market economies, monetary and fiscal policies go through unending cycles of experimentation, effectively crimping growth and causing dislocations in the economy. The first place those signs are noticed is in the stock market. Indeed, Bjørnland and Leitemo (2009) document the interdependence between macroeconomic performance and the stock market. A soft economy prompts global portfolio investors to damp their holdings of financial assets and sparking market gyrations. In the ensuing volatile environment, a country's currency weakens as investors offload their market holdings and capital exits the economy. The situation is worsened if the country's central bank lacks the firepower with adequate reserves to stem the falling local currency.

Again, most of the sub-Saharan African exchanges have so many of their listed firms in the services and commodities sectors. These firms are easily buffeted by global demand shocks beyond the control of their respective countries. The shocks in the underlying economies are transmitted directly to the stock markets as switching regimes in the level of returns on investments.

Also, the activities of global rating agencies like S&P, Fitch and Moody's add to the market turbulence in sub-Saharan Africa. The investment mandates of most of these global investing firms prohibit investments in assets with junk rating status. Also, a key requirement of investors in a country's stock market is its membership of global financial indices like the MSCI, FTSE Frontier Index, NASDAQ Emerging Markets Index, among others. As economic fortunes ebb and wane in response to economic conditions in countries, so does the risk of the economy's sovereign rating migrating over time. In response to this, investors rebalance their portfolios taking into account the risks in the market. This results in further unsettling of the financial markets. Evidence of this behaviour by global investors is provided in Della Croce, Stewart and Yermo (2011). Other researchers like Shleifer and Vishny (2011) document extensively instances where, for example, investors resorted to fire sales in situations of sovereign rating migrations to lower levels prompting a surge in volatility. In the face of all these developments, risk, being an artificial construct, should reflect fully the underlying behaviour of the economy that gives rise to the data. That is where researchers can potentially give the full narrative of the behaviour of market data.

Regime Switching Versus Non-Switching Volatility Models

The basis of regime switching models is relatively simple. Financial data are generated by underlying activities in an economy. Such activities undergo phases of booms, busts and everything in-between. Volatility, being central to finance, should reflect this multi-state characteristic in a fine-grained manner. Hamilton's work (1989) on the business cycles is premised on this nonlinearity. Later authors like Luo, Seco, Wang and Dash Wu (2010), Haas, Mittnik and Paolella (2004) and Cai (1994) confirmed the superiority of regime switching over non-switching models, claiming they are more accurate in capturing the changing expectations, variances and correlations over time.

Ardia, Bluteau, Boudt and Catania (2018) undertook a large-scale study of various heteroscedastic functions employing stocks, aggregate market indices and currency pairs of the developed markets and found that regime switching volatility models performed better in terms of model fit of the data by essentially capturing the nonlinear irregularities that characterises financial market data. Financial market data is a reflection of the behaviour of real people and institutions responding to monetary and fiscal policies and adjusting their decisions in the light of changes in these variables for optimum outcomes. The heterogeneity of these actions is best captured with piecewise GARCH models. Thus, the (G)ARCH models of Engle (1982) and Bollerslev (1986) are best seen as "volatility primitives", the basis of building complex and realistic models.

The methodological basis for analysing market data using regime switching models is built on sound theoretical footing (Diebold, Lee, & Weinbach, 1994). Various authors have also recommended regime switching models in place of non-switching in modeling heteroscedastic outcomes. Lamoureux and Lastrapes (1990) noted the shortfalls in volatility models especially where and when they are needed most – during financial crises when tails risks should be measured accurately. During this time, single-regime models undershoot risks and lull investors into complacency until the risks manifest. That is the position of Danielsson (2002) who posited that financial data is endogenous to the underlying developments driving asset prices in the markets. It is an attribute missing in non-regime-switching models which as a result lack the robustness to describe real markets and provide adequate guidance for risk-based investing and decision making. Eichengreen and Bordo (2002) observed that there is increased frequency of crises in financial markets today. Danielsson (2008) again takes issue with the models, observing that these models fail when they are needed most. It would appear these models are built without due cognisance of the environment generating the data to carefully look at the changing stylised characteristics such as the abrupt switching of the levels of returns or the quirky but brief extreme returns that lead to heavy-tailed distributions observed in financial time series with increasing frequency.

In this environment, improving the models by aligning them closely with the data generation environment for estimating the volatility models is going to be an important modelling technique to forestalling or minimising the impact of crises such as the recent global financial crisis stemming out of mispricing of risks in the US subprime mortgage sector.

Non-switching volatility models adapt to changes in market conditions rather slowly. On the other hand, several studies to do with extreme risk have shown that regime switching models are responsive to the changing states of the return generation process, communicating to users the resulting rich heteroscedastic dynamics for decision making (Miao, Wu, & Su, 2013; Weron, Bierbrauer, & Trück, 2004; Kawata & Kijima, 2007). The behaviour of the stock market during a recession vis-a-vis expansion in an economy is the clearest evidence of the presence of market regimes. Standard GARCH models do not staircase risks across time, effectively discounting the evidence based on the behaviour of equity returns in periods of recessions and expansions respectively. During recessions, nervous investors flee the equity markets to the safety of sovereign instruments. The reverse is seen in economic expansions. Thus, characterising the risks in the financial markets cannot be divorced from the underlying dynamics at play in the economy especially in measures of volatility, a pure financial construct for risk. The continuing use of the single GARCH models can no longer be justified based on the increasingly turbulent market in a nonlinear environment with many financial products trading at so much speed under a multiplicity of contracts. At best, these earlier GARCH models should be seen as primitives needed to build much more realistic market artefacts that can stand the increasingly fast and unstable real financial markets.

For macroeconomics, regime switching now finds use in the policy toolbox of central banks. Alba and Wang (2017), and Duffy and Engle-Warnick (2006) employed Markov regime switching in studying the applicability of the Taylor rule to monetary policy and how it affects GDP growth and dictates inflation outcomes in the US economy. Indeed, Hamilton (2016) makes the case for the incorporation of changes in regimes in macroeconomic planning citing instances of booms and busts as examples of regime switching mirroring underlying developments in the economy. There is evidence that this is the case in modern macroeconomic policy: inflation targeting (Claeys, 2015), exchange rate policy (Fiess & Shankar, 2009), general monetary policy (Owyang & Ramey, 2004), taxation and debt (Davig, 2004), among many studies in macroeconomics.

The level of foreign exchange exerts a lot of control on the financial markets of developing countries (Bosworth, Collins, & Reinhart, 1999).

Hauser, Marcus and Yaari (1994) discussed at length the additional cost of hedging foreign currency risk by investors trading in markets prone to exchange instability. This prompted Chkili and Nguyen (2014) and Walid, Chaker, Masood and Fry (2011) to investigate the issues at the nexus of equity market returns and foreign exchange rates using multi-state heteroscedastic models to study how spikes in the foreign exchange markets spill into the stock markets in emerging economies. The authors relied on the flexibility afforded by regime switching models, taking advantage of their adaptation to nonlinear environments of evolving uncertainty in the foreign exchange markets. Thus, there is a lot in monetary and fiscal policy planning using regime switching models for policymakers.

Independent of the above factors, leverage in financial markets do cause heavy-tails now and then and by extension introduce time-varying parameters of the DGP leading to observed switches in the distributions (Thurner, Farmer, & Geanakoplos, 2012). There is ample evidence in the extant literature, for example, Guidolin (2011), Diebold and Inoue (2001), and Cont (2007), that heavy-tails in financial time series induce surges in volatility leading to regime changes. These effects, usually severe in the left tails, are laden with sudden risks and had led to losses or perhaps the collapse of firms as well documented in finance (Allington, McCombie, & Pike, 2012; Stoyanov, Rachev, Racheva-Yotova, & Fabozzi, 2011; Hilal, Poon, & Tawn, 2011; Chincarini, 2007). Heavy- or fat-tailed distributions connoting distributions with far more probability mass in the tails is much more common in financial data and understanding the nature of how these is another layer of risk provides valuable insights into what causes extreme volatility in the markets. A typical manifestation of this is the volatility clustering empirically observed in market returns. The fat-tails are a property driven largely by the extreme return outcomes and this endows the series with large kurtosis beyond values characterising normal or Gaussian distributions. It is the main property that leads to the deviations of normality observed in many financial time series (Allington, McCombie, & Pike, 2012). Frontier markets are a step away from emerging market status; thus, they share some market characteristics in terms of leverage effects (Ibragimov, Ibragimov, & Kattuman, 2013). Basically, it has been observed that leverage effects become dominant in the market when equity prices start to fall. Since a firm debt is fixed, a falling equity means debt is becoming prominent; hence risky. This sends up volatility seen as clustering in the time series of returns (Daal, Naka, & Yu, 2007).

Again, heavy-tails in frontier markets could be due partly to hidden information effects related to trading of financial assets. The long memory effects observed in some frontier markets in African, namely, Botswana, Ghana, Mauritius, and Namibia by Rambaccussing (2010) could very much be a function of the latency of information flow in the assets and how this affects the frequency of trading. The halting nature of trade as investors discover information with difficulty in these markets suddenly sends equity and indeed all asset prices up or down. This shows in the form of a lower or higher volatility in the markets and registers as heavy-tails.

The Case for Regime-switching in Volatility Modeling for Sub-Saharan Equity Returns

For long, it has been known in the investor community that sub-Saharan Africa frontier equity markets present investors with opportunities to diversify their portfolios and also make rich pickings in investment. The low correlation has been attributed to lack of cross-listing of firms across these markets (Bahlous, 2013). Serkin (2015) made the point that portfolio managers who are able to navigate the information starved sub-Saharan frontier stock markets have made decent returns for their investors. Fact remains, however, that academic research has not gone beyond the single regime volatility models needed to provide guidance to investors in such a turbulent part of Africa.

Across all of sub-Saharan Africa, the economies are prone regularly to bouts of economic slowdown, foreign currency problems, political uncertainty and unrest, and monetary and fiscal policy missteps. These are the underlying conditions that induce nonlinearities in financial data. The incorporation of regime switching to capture these multi-state evolving uncertainties therefore should be natural in volatility modelling. Yet there is very little in the extant literature on volatility modeling adopting this approach in sub-Saharan frontier equity markets. Outside sub-Saharan Africa, academics have recognised the nature of these peculiarities in their economies and the data generated; thus, volatility modeling in these regions does incorporate regimes. In the Gulf and North African region, Balcilar et al. (2013) have incorporated regimes in studying investor herd behaviour on the Gulf Arab bourses. Work done by Khalifa, Hammoudeh and Otranto (2014) looked at the spillover effects of the recent US financial crisis on emerging markets using regime switching. Charfeddine and Ajmi (2013) employed regime switching to study long memory effects in market returns on the Tunisia bourse. In Islamic finance, regime switching models have been employed in the study of the sukuk and sharia equities (Aloui, Hammoudeh, & Hamida, 2015).

Unfortunately, there is a lack of regime switching model characterisation in the equity markets of the sub-Saharan African region in the academic literature. This is contrary to the evidence in the empirical data. Volatility models are limited to non-switching GARCH models whose limitations are known to researchers. Adjasi (2009), Nyamongo and Misati (2010), Chinzara and Slyper (2013), Uyaebo, Atoi and Usman (2015), Carsamer (2016) among many researchers have studied volatility of returns of equities in the sub-region employing macroeconomic variables in the context of single-regime models. None has looked at the breaks in the continuity of the data as a result of switching occasioned by the underlying developments which affect asset prices on the exchanges. This is the point where this work distinguishes itself from that of the earlier researchers. Regime switching models contrast sharply the assumptions of linearity in the DGP and clearly take advantage of the stylised facts of financial data.

In short, when it comes to the trading environment, regime switching models, whether for returns or for volatility, provides a complete mapping of the developments in the underlying economy to the risks of returns by calibrating risk levels to the potential innovations or shocks arising from policy imperatives and disturbances due to economic or political changes with direct bearing on the financial markets of a country.

Chapter Summary

Modelling market risk is important for trading in securities. Unfornately, it is easy for the models to be misspecified and by so doing fail to properly capture the risks associated with trading and investment decisions-making. This chapter reviewed the seminal and current thinking about risk and volatility models. The industry workhorse has always been the classical GARCH and its variants. The chapter pointed out the deficiency of these GARCH models and reviewed the regime switching models which are better at modelling the market risks based on the underlying developments driving the returns in the markets.

CHAPTER THREE

METHODOLOGY

In this chapter, the methodology of the various regime switching models including heavy-tailed distributions are reviewed. The specification adopted is the GARCH model and endow it with changes in regimes of the equity returns. Complex statistical models are known to overfit the data. We therefore look at backtesting these complex models with value-at-risk models, ensuring we have well fitted volatility functions to the data.

Introduction

Regime switching volatility models are able to accommodate the changing levels of heteroscedasticity found in market returns, especially where the underlying unobserved stochastic process exhibit nonlinearities in the data. It takes a piecewise approach that captures regime-by-regime, the stylized facts in financial time series unlike non-switching GARCH models that average the volatility across all time periods. Typically, sub-Saharan African equity trading is characterised by thin and asynchronous trades mirroring developments in the underlying economies. For such economies in constant flux, time-varying heteroscedastic functions with regime switching are the appropriate models that reflect the endogenous dynamics generating the data and provide a better fit to the returns. As rightly pointed in Marccuci (2005), switching models compared to single regime GARCH functions are able to adapt to the changing conditions in the market, thus offering investors a warning against excessive and disruptive market swings.

In this chapter, therefore, we specify Markov regime switching volatility models that will be used to fit the data in the next chapter. The models are considered within the univariate financial time series framework.

Regime-switching GARCH Models

Let $\{r_t\}_{t=1}^{T}$ be a time series of sample returns where $t = 1, 2, . . ., T$. We suppose that these series exhibit all the stylised properties of financial time series observed in Cont (2001). We adopt a Bayesian inference approach instead of maximum likelihood in estimating the parameters of the model for two principal reasons. First, the problem of estimating parameters for regime switching models is prohibitively expensive or almost intractable. Gray (1996) demonstrated that for a model of k regimes involving T observations, we need to integrate over K^T paths. The solution to this path-dependency problem is the Bayesian method which has been demonstrated elegantly in Haas, Mittnik and Paolella (2004) who adopted a regime-by-regime GARCH approach to modeling each regime specific parameters. Secondly, Bayesian estimation accommodates parameter uncertainty, a process that minimises model risk (Clyde & George, 2004). We adopt the specification of the model in Haas, Mittnik and Paolella (2004). In general, we state the distribution assuming regime switching governed by:

$$r_t | (S_t = k, F_{t-1}) \sim \wp(\mu_{k,t}, q_{k,t}, \eta_k), \qquad (3.1)$$

where $\wp(\mu_{k,t}, q_{k,t}, \eta_k)$ is the probability model describing the complete regime k behaviour of the returns r_t with mean $\mu_{k,t}$ and scale $q_{k,t}$ at time t. η_k is the guiding parameter(s) describing the shape of the distribution of returns. Generally, $\mu_{k,t}$ is taken to be practically zero in the long run for the return series or if not zero, then the series has to be de-meaned. It must be emphasized that the parameters in the probability model (3.1) are regime dependent. The information contained in the event history up to and including time $t-1$ is denoted F_{t-1}. The variable S_t is the state space with a finite set of values $k = 1, 2, 3, . . ., K$, which evolve according to a Markov governing law given by:

$$\mathbb{P}(S_t | S_{t-1}, S_{t-2}, . . .) = \mathbb{P}(S_t | S_{t-1}) \tag{3.2}$$

The multi-state transition of the volatility from state m to state n is driven by a Kx K first-order homogenous stochastic matrix. This stochastic matrix transiting the regimes from state to state can be written out as:

$$\mathbb{P}(S_t = n | S_{t-1} = m) = \begin{bmatrix} p_{11} & \cdots & p_{1n} \\ \vdots & \ddots & \vdots \\ p_{m1} & \cdots & p_{mn} \end{bmatrix} \tag{3.3}$$

where p_{mn} is the probability of transitioning from regime m to regime n in the period t which is defined by:

$$p_{mn} = \frac{\exp(X\beta_{mn})}{1+\exp(X\beta_{mn})} \tag{3.4}$$

This matrix has all the properties of a typical Markov chain such that each element $p_{ij} \geq 0$ for all $i, j \in \{1, 2, . . ., K\}$ and again for each row, $\sum_{j=1}^{K} p_{ij} = 1 \ \forall i \in \{1, 2, . . ., K\}$. By construction, each row of the transition matrix $\mathbb{P}$ is an approximation of a probability density functions over the given state K.

Conditional Heteroscedastic Functions

The variance function, $q_{t,k}$, in (3.1) is a piecewise GARCH-type function which depends on a given lagged error specification, ε_{t-1}^2, of the residual error distribution, the past variance $q_{t,k-1}$ and a vector of parameters η_k which are regime-dependent. This function takes the form

$$q_{t,k} = q(\varepsilon_{t-1}, y_{t-1}, \eta_k) \tag{3.5}$$

The variance function $q_{t,k}(.)$ in most GARCH specifications is constrained such that $q_{t,k}(.) > 0$ and its parameters depend on the regime s_k of the market returns.

A key property of $q_{t,k}(.)$ is its ability to capture important empirical variance-related stylised properties of a return series. Originally, Engle's ARCH (1982), later generalized into GARCH by Bollerslev (1986) was meant to capture volatility clustering in a return series. Volatility clustering is the tendency to observe clusters in return series over time, that is, volatile periods tend to follow volatile times and vice-versa. Other important effects include asymmetry in response to bad news compared to good news (Glosten, Jagannathan, & Runkle, 1993) and the leveraged effect originally observed by Black (1976) and formalised into a heteroscedastic function by Nelson (1991). Thus, the various GARCH model specifications in the finance literature reflect some important property of financial time series (For a comprehensive review of GARCH models, see Teräsvirta (2009), Poon and Granger (2005), Engle (2001) and Franses and Van Dijk (1996). For this study, consideration is given to the regime-switching versions of GARCH (1, 1), the exponential GARCH (1, 1) and the GJR-GARCH (1, 1). The choice of lower order versions of these volatility models is based on research in the econometric literature showing that higher order heteroscedastic functions do not necessarily outperform lower order models (Hansen & Lunde, 2005).

The Regime-switching GARCH (1, 1) Model

The regime switching GARCH (1, 1) model is specified as follows:

$$q_{t,k} = \omega_k + \alpha_k \varepsilon_{t-1}^2 + \beta_k q_{k,t-1} \tag{3.6}$$

where

$$\varepsilon_t \sim iid\mathfrak{D}(0, \sigma_\varepsilon^2) \quad (3.7)$$

The vector of regime-dependent parameters $\varphi_k = \{\omega_k, \alpha_k, \beta_k\}$ is to be estimated. We constrain the parameters as $\omega_k > 0$, $\alpha_k, \beta_k \geq 0$ and $\alpha_k + \beta_k < 1$, guaranteeing that the variance $q_{t,k}$ will be positive and exist for a particular regime k. $\mathfrak{D}(.)$ is refers to identical and independent random variables drawn from a normal, student-*t*, skewed student-*t* or some other type of distribution which describes the tail behaviour of the returns.

The Regime-switching GJR-GARCH (1, 1) Model054 027 3033

The GJR-GARCH (Glosten, Jagannathan, & Runkle, 1993) is an asymmetric GARCH model with an extra parameter to take care of the leverage effects induced in the data when the value of equity falls relative to the market value of debt. Falling equity prices lower returns and increase volatility in financial markets. Black (1976) summarised this as "*a drop in the value of the firm will cause a negative return on its stock, and will usually increase the leverage of the stock. [...] That rise in the debt-equity ratio will surely mean a rise in the volatility of the stock*". Christie (1982) provides additional support for this observation of how tumbling stock prices elevate volatility as a result of the rise in the debt/equity ratio in the firm's capital structure. To accommodate this behaviour, the GARCH (1, 1) in (3.6) is re-specified as:

$$q_{t,k} = \omega_k + \alpha_k \varepsilon_{t-1}^2 + \gamma \mathbb{I}_{\{\varepsilon_{t-1}<0\}} \varepsilon_{t-1}^2 + \beta_k q_{t-1} \quad (3.8)$$

$$\varepsilon_t \sim iid\mathfrak{D}(0, \sigma_\varepsilon^2) \quad (3.9)$$

The indicator $\mathbb{I}_{\{.\}}$ takes a value of 0 or 1 depending on the outcome of the binary condition $\varepsilon_{t-1} < 0$. The rest of the symbols have their usual meanings.

The Regime Switching Exponential GARCH Model

The exponential GARCH (1, 1) (EGARCH) was pioneered by Nelson (1991). It addresses two issues in volatility modeling. Classical GARCH assumes it is the magnitude of the information content in the news reaching the market that determined the volatility. Observations in the market however point to the market reacting asymmetrically to news. Reaction of financial markets to positive news on one hand, and negative developments move volatility unequally. Markets swing much more intensely with the arrival of bad news compared to the receipt of good news of the same magnitude.

Secondly, the EGARCH seeks to relax the restrictions on the volatility parameters. This is a contentious point in the volatility modeling literature. Alexander (2008) claims that imposing restrictions on optimisation routines probably implies the model is mis-specified. This motivated Nelson (1991) to specify the EGARCH as log variance instead of the variance. This ensures positivity of the variance without imposing any restrictions on the parameters. The regime-switching EGARCH is stated as follows:

$$In(q_t) = \omega_k + \alpha_k(|\varepsilon_{t-1}| - E[|\varepsilon_{t-1}|]) + \gamma\varepsilon_{t-1} + \beta_k In(q_{t-1}) \qquad (3.10)$$

$$\varepsilon_t \sim iid\mathfrak{D}(0, \sigma_\varepsilon^2) \qquad (3.11)$$

There are no restrictions on the parameters. The term $\alpha_k(|\varepsilon_{t-1}| - E[|\varepsilon_{t-1}|])$ represents the magnitude of information in the news. We require each of the regimes of the distribution to be piecewise stationary. This is achieved when $\beta_k < 1$. The asymmetric response to news arrival in the market induces leptokurtic tails in the return distribution which the EGARCH captures in addition to volatility clustering.

A related concept from Bayesian analysis is that of the posterior mean. The posterior mean is the weighted average of the data and the prior. If this posterior mean is updated sequentially as the daily return becomes available, we get an update of the changing regime that the data is undergoing and the probabilities involved. This reflects the underlying changes in the economy driving the market returns.

Conditional Distributions of the Errors

The conditional distributions of the errors denoted by $\mathfrak{D}(.)$ in each of the heteroscedastic specifications for the GARCH models in (3.7), (3.9) and (3.11) determine the tail behaviour of the distribution in each regime. To avoid complexity in the specification, we specify the same conditional distribution of errors in each regime. Among the specifications encountered in the volatility literature are the normal, student-t, generalized error distributions (GED) and their skewed versions.

Normal Distribution

The probability density function (pdf) of the standard normal distribution of the errors is given by

$$f(\varepsilon|\mu,\sigma) = (2\pi)^{-1/2}\exp\left(-\frac{\varepsilon^2}{2}\right) \text{ for } -\infty < \varepsilon < \infty \tag{3.12}$$

where $\mu = 0$ and $\sigma = 1$. The Gaussian distribution is described completely by its mean and variance. It has been used less in volatility modeling in the present time than in the past. Return innovations have been found to be non-Gaussian across different types of markets and asset classes; hence its discontinuance in modelling the tails of returns (Curto, Pinto, & Tavares, 2009; Ushad, Fowdar, Vinesh, & Jowaheer, 2008; Gray & French, 1990).

Student-*t* Distribution

The student-*t* distribution finds its use in the modeling of distributions with fat-tails. The data from frontier equity markets exhibits some anomalous properties, particularly heavy-tails, which makes this conditional distribution a good fit. If X is drawn from a Student-*t* distribution written as $X \sim t_{\nu}(\mu, \sigma)$, then the pdf for a realization ε is written as:

$$f(\varepsilon|\nu) = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi}\Gamma(\frac{\nu}{2})}(1 + \frac{\varepsilon^2}{\nu})^{-(\nu+1)/2} \quad (3.13)$$

where $\Gamma(.)$ is the gamma function and ν, which takes the form:

$$\frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi}\Gamma(\frac{\nu}{2})} = \begin{cases} \frac{(\nu-1)(\nu-3)\ldots5\,.3}{2\sqrt{\nu}(\nu-2)(\nu-4)\ldots4\,.2}, & \nu\ even \\ \frac{(\nu-1)(\nu-3)\ldots4\,.2}{\pi\sqrt{\nu}(\nu-2)(\nu-4)\ldots5\,.3}, & \nu\ odd \end{cases} \quad (3.14)$$

where ν is the *degrees of freedom*. The distribution is symmetrical about zero and has the support $-\infty < t < \infty$. It is to be noted that the distribution of the student-*t* as shown in equation (3.14) is independent of the mean μ and the standard deviation σ.

The GED Distribution

The GED distribution is another relation used in modeling heavy-tails. Used by Nelson (1991) in studying the tail behaviour of returns in finance, it fits flexibly the distributions of the innovations and provides an alternative to the student-*t* function in financial modeling where extreme outcomes are possible (Fan, Zhang, Tsai, & Wei, 2008). It is described by the pdf

$$f(\varepsilon|\nu) = \frac{\nu.\exp\{-\frac{1}{2}|\frac{\varepsilon}{\lambda}|^{\nu}\}}{\lambda 2^{1+\frac{1}{\nu}}\Gamma(\frac{1}{\nu})}, \ \nu > 0, \ \varepsilon \in \mathbb{R} \quad (3.15)$$

where $\lambda = \left[\frac{1}{2^{\frac{2}{\nu}}}\frac{\Gamma(\frac{1}{\nu})}{\Gamma(\frac{3}{\nu})}\right]^{1/2}$ is the tail thickness parameter controlled by the degrees of freedom ν and $\Gamma(.)$ is the usual gamma function. The GED subsumes most of the popular innovations in use. For example, when $\nu=1$ we have the Gaussian distribution, and the double exponential distribution when $\nu=2$. It is also used in modeling thin-tailed phenomena like the uniform distribution over some specific interval (Diongue, Guegan, & Wolff, 2010).

Parameter Estimation for Regime Switching Models

The estimation of the volatility model takes place via Markov chain Monte Carlo (MCMC) techniques within the Bayesian modelling paradigm. This is an evaluation of the prior and likelihood to find the posterior from which the estimators of our parameters come from. We resort to Bayesian methods to overcome the path-dependency problem identified by lots of authors in volatility modeling (Billio & Cavicchioli, 2017; Bauwens, Preminger, & Rombouts, 2010; Chuffart, 2015; Abramsom & Cohen, 2007). With ARCH heteroscedastic functions, the volatility is a function of past errors. However, GARCH volatility models are recurring functions of lags of both past errors and variances. This makes the variance at time t a function of past variances at time $t-1$, $t-2$, $t-3$, etc. Evaluating the likelihood with a memory of its past history in the case of regime-switching is therefore not feasible as demonstrated in Abramson and Cohen (2007).

Cai (1994) and Hamilton and Susmel (1994) avoided this problem by modelling regime-switching volatility using the ARCH model. So, the problem with using regime switching GARCH models which as Bollerslev (1986) showed are parsimonious remained intractable until the pioneering

$$f(r_t|\varpi, \mathcal{F}_{T-1}) = \sum_{m=1}^{K}\sum_{n=1}^{K} p_{mn}\mathcal{g}_{i,t-1} f_{D(.)}(r_t|s_t = n, \varpi, \mathcal{F}_{T-1}) \quad (3.20)$$

In (3.20), $\mathcal{g}_{i,t-1} = P[s_{t-1} = m|\varpi, \mathcal{F}_{T-1}]$ is the history of returns in state i at time $t-1$.

From (3.16), applying the total probability to the prior, we obtain

$$p(\varpi|\mathcal{F}_{T-1}) = p(\omega)p(\alpha|\omega)p(\beta|\omega,\alpha)p(\sigma|\omega,\alpha,\beta)p(\nu|\omega,\alpha,\beta,\sigma)p(\pi|\omega,\alpha,\beta,\sigma) \quad (3.21)$$

Using the assumption of independence of the parameters, (3.21) becomes

$$p(\varpi|\mathcal{F}_{T-1}) = p(\omega)p(\alpha)p(\beta)p(\sigma)p(\nu)p(\pi) \quad (3.22)$$

There is a plethora of options when it comes to choosing the prior distribution. Lemoine (2019), Gelman, Simpson and Betancourt (2017), Gelman (2006), Polson and Scott (2012), Kass and Wasserman (1996), Berger, Bernardo and Sun (2015), among others, provide guidelines on the appropriate choice of priors encountered in Bayesian inference. In particular, Kim and Nelson (1999) and Das and Yoo (2004) used normal priors for the GARCH coefficients, ω, α and β. The variance has to be constrained such that $\sigma_k^2 > 0$. We adopt the approach of Ardia and Hoogerheide (2010) in using inverse-gamma distribution with shape and scale parameters respectively a and b as

$$\sigma_k^2 \sim InverseGamma(a, b) \quad (3.23)$$

Unfortunately, the nature of the posterior in (3.22) is unknown. For problems of this nature, various writers, for example, Sherlock, Fearnhead and Roberts (2010) have recommended the use of the random walk Metropolis-Hastings algorithm (Hastings, 1970; Metropolis, Rosenbluth, Rosenbluth, Teller, &

work of Gray (1996) who used Gibbs sampling to simulate the posterior from which the needed GARCH parameters were estimated. Later, Augustyniak (2014) and Haas, Mittnik and Paolella (2004) approached this problem using other Monte Carlo simulation methods with improved model fit without the constraints of tractability. This is the approach we use in estimating the parameters of the regime switching GARCH model in this work.

Approach to Parameter Estimation via MCMC

Given the parameter vector $\boldsymbol{\varpi} = \{\omega, \alpha, \beta, \sigma_k, \nu_k, \pi_{mn}\}'$ where ω, α, β are state or regime dependent GARCH regression coefficients, σ_k the *k*-regime dependent volatility, ν_k is the tail thickness parameter and π_{mn} the transition matrix from state m to state n under a Markov transitioning kernel. Invoking Bayes' probability rule given the filtration $\mathcal{F}_{T-1}$ at a point *t-1*, we have

$$p(\varpi|r_t, \mathcal{F}_{T-1}) = \frac{p(\varpi|\mathcal{F}_{T-1})p(r_t|\varpi, \mathcal{F}_{T-1})}{p(r_t)} \qquad (3.16)$$

Simplifying (3.16), we get

$$p(\varpi|r_t, \mathcal{F}_t) = \kappa p(\varpi|\mathcal{F}_{T-1})p(r_t|\varpi, \mathcal{F}_{T-1}) \quad (3.17)$$

In Equation (3.17), κ is a normalizing constant equal to $\frac{1}{p(r_t)}$ which ensures the pdf function is properly defined. In Equation (3.17), $p(\varpi|r_t, \mathcal{F}_{T-1})$ is the posterior, $p(\varpi|\mathcal{F}_{T-1})$ is the prior and the likelihood is $p(r_t|\varpi, \mathcal{F}_{T-1})$. Since κ is a normalizing constant, Equation (3.17) can be written as

$$p(\varpi|r_t, \mathcal{F}_t) \propto p(\varpi|\mathcal{F}_{T-1})p(r_t|\varpi, \mathcal{F}_{T-1}) \quad (3.18)$$

Assuming independence, the likelihood can be written jointly as

$$p(r_t|\varpi, \mathcal{F}_{T-1},) = \prod_{t=1}^{T} f(r_t|\varpi, \mathcal{F}_{T-1}) \qquad (3.19)$$

The dynamics of the pdf $p(.)$ can be written thus

Teller, 1953), a subclass of the Markov Chain Monte Carlo algorithms for generating samples from the posterior.

The Random Walk Metropolis-Hastings Algorithm

Let the proposal density conditional on x be $w(y|x)$ and defined as

$$y = x + \epsilon \tag{3.24}$$

with ϵ having a distribution $h(.)$. We assume that $h(.)$ is symmetric about 0.

If we take the mean conditioned on x, we get

$$w(y|x) = h(\epsilon) \tag{3.24}$$

and

$$w(x|y) = h(-\epsilon) = h(\epsilon), \tag{3.25}$$

since it is symmetric.

We define the acceptance ratio for the Metropolis-Hastings algorithm as

$$d(y|x) = \min\left\{1, \frac{\pi(y)w(x|y)}{\pi(x)w(y|x)}\right\} \tag{3.26}$$

Based on the assumption of symmetry implied by (3.24) and (3.25), $w(y|x) = w(x|y)$. Therefore, (3.26) becomes

$$d(y|x) = min\left\{1, \frac{\pi(y)}{\pi(x)}\right\} \tag{3.27}$$

Hence, given the state-space, if we start from x, to transition to another state, we proceed using these steps:

(i) Simulate $\epsilon \sim h(.)$ and set $y = x + \epsilon$.

(ii) Compute $d(y|x) = min\left\{1, \frac{\pi(y)}{\pi(x)}\right\}$

(iii) Simulate $u \sim Uniform(0, 1)$

(iv) If $u \leq d(y|x)$, we accept y as the next state, else we stay at x.

The advantage of the random walk Metropolis-Hastings routine is that it does not need tuning. This feature makes for faster convergence and gives it such attraction in simulation (Chib & Greenberg, 1995).

Model Diagnostics

Complex statistical models with lots of parameters have been known to overfit the data. Although regime switching volatility models with their increased parameters gives them flexibility to adapt to changing market conditions, they are complex. We need to backtest them to ensure the nature of the observed volatility in the markets reflects the DGP and are therefore accurate for predictive decision-making purposes. Indeed Gelman, Stern, Dunson, Vehtari and Rubin (2013) posit that models can fail to '*reflect the process that generated the data in any number of ways. . .*' (pg. 148). The consensus in finance against overfitting therefore is backtesting to validate the models against historical data.

The pioneering work of Kupiec (1995), Christoffersson (1998) and Engle and Manganelli (2004) is set on this simple idea that consecutive breaches of a given loss threshold predisposes firm to bankruptcy. This is where the model should be good at picking. The process therefore estimates both conditional and unconditional coverage tests of the correct number of exceedances of the thresholds established by the risk models. The Kupiec point-of-proof test (POF-test) establishes whether the frequency of exceptions over a given time period are in line with our level of chosen confidence. Coverage tests, on the other hand, tests whether the DGP changes over time. One such approach is the Christofferson test. These tests rely on the value-at-risk (VaR) to make these determinations. The VaR is defined as:

$$VaR_\alpha^T = -\mu - \sigma F^{-1}(\alpha) \quad (3.28)$$

where $F^{-1}(.)$ is the inverse cumulative density function evaluated in the left tail given a designation α. VaR is usually estimated over a given time horizon T.

It has been shown by Campbell (2005) that tests which accommodate several quantities are robust in discriminating good risk models from bad risk models (for a comprehensive review of such tests see Abad, Benito & López (2014)). One such test is the Dynamic Quantile (DQ) test of Engle and Manganelli (2004). The DQ test proposes a linear regression which examines jointly whether the VaR forecasts by the models give correct unconditional coverage and if the exceedances of the VaR are independent over time. The independence of the VaR forecasts is essential to avoid clustering.

Engle and Manganelli (2004) specified the generic regression equation as follows:

$$VaR_t(\beta) = \beta_0 + \sum_{i=1}^{q} \beta_i VaR_{t-i}(\beta) + \sum_{j=1}^{r} \beta_i l(r_{t-j}) \quad (3.29)$$

with $p = q + r + 1$ the dimension of β, $l(.)$ is the number of observable lags in the autoregressive function and r_{t-j} the returns from the financial asset. The DQ test statistic has an asymptotic χ_k^2 distribution defined as

$$DQ = \frac{\hat{\beta}' r\prime r \hat{\beta}}{p(1-p)\prime} \quad (3.30)$$

where $\hat{\beta} = (r'r)^{-1} r'Y$ denotes the ordinary least squares estimator.

Heavy-tail Distributions

Tail distributions of assets returns show marked deviations from normality. Usually, the tails are characterised by heavy-tailed phenomenon obeying some form of Pareto-law of the form:

$$F(x) = Ax^{-\alpha} \tag{3.31}$$

In general, the nature of the tail distributions is classified into three - thin-, light- and heavy-tails and are characterised Weibull, Gumbel and Frechet distributions, respectively. The characterisations stem from the Fisher–Tippett–Gnedenko (Gnedenko, 1943; Fisher & Tippett, 1928) theorem as follows:

Let the series of independent and identically distributed returns be $\{r_i\}_1^T$ and $M_T = \max\{r_1, . . ., r_T\}$. The distribution of M_T is given by:

$$\lim_{T \to \infty} P\left\{\frac{M_T - a_T}{b_T} \leq x\right\} = F(x) \tag{3.32}$$

with the constant a_T and $b_T > 0$ which must exist are respectively, $a_T = TE[r_1]$ and $b_T = \sqrt{V[r_1]}$. In the limit, equation (3.32) becomes: $F_\xi(x) =$

$$\begin{cases} \exp\{-(1+\xi x)^{-\frac{1}{\xi}}\}, & \xi \neq 0 \\ \exp(-\exp(-x)), & \xi = 0 \end{cases} \tag{3.33}$$

In equation (3.33), $F_\xi(.)$ assume a Frechet distribution if $\xi > 0$; a Weibull if $\xi < 0$ and Gumbel if $\xi = 0$. The ξ is the tail index and for financial returns, the extreme distributions are heavy-tailed; hence $\xi > 0$ (Fama, 1963).

Several approaches due to Hill (1975), Pickands (1975), Beirlant, Vynckier and Teugels (1996), Kratz and Resnick (1996), Feuerverger and Hall (1999), among many schemes exist in the extant literature for estimating the tail index. Hill (1975) is much used because of its lack of assumptions about the distribution of the data coupled with its simplicity. It is estimated as:

$$\xi_{k,T} = \{k^{-1}\sum_{i=1}^{k}(Inr^{(i)} - Inr^{(k+1)})\}^{-1}, \text{ for } 1 \leq k < T \qquad (3.34)$$

where the variables have been previously defined.

Chapter Summary

Regime switching risk models are endowed with a lot of parameters which make for their flexibility in fitting the nonlinearities in market returns data. However, this flexibility as a result of the number of parameters makes estimation difficult due to the path-dependency nature of the evolution of volatility particularly with respect to GARCH models. This explains the reliance on MCMC simulation methods. Added to the regime switching, is the persistence of fat-tailed distribution that induces another layer of uncertainty to the correct estimation of the risk measures in the frontier equity markets. What is not clear is the level of dominance or interplay of these two market concepts in determining correctly the surges in volatility in the trading environment.

In this chapter, we laid out the theoretical basis for the use of the random walk Metropolis-Hastings scheme which is part of the MCMC algorithms used in sampling from a posterior distribution for parameter estimation. At the same time, we note that such complex statistical models have the tendency to overfit the data and give poor forecasts. This is particularly true for risk models as pointed out in Salmon (2012), Danielsson (2008) and Danielsson (2002). The standard industry practice to guard against such poor risk models is to backtest them against historical data. We laid out the criteria and methods to be used on backtesting the models in the upcoming chapter.

CHAPTER FOUR

RESULTS AND DISCUSSION

Introduction

This chapter analyses the data by running 144 heteroscedastic models incorporating single and double regimes making use of normal, student-*t* and GED with their respective skewed versions. Model convergence is made using relative numerical efficiency of the estimates. Model fit is assessed with Deviance Information Criteria (DIC). To ensure the models are not overly complex, they are backtested using the Dynamic Quantile (DQ) regression tests. The appropriate models describing the evolutions of volatility in the respective markets is determined and based on the estimates, we described and plotted the patterns of low- and high-risk regimes using smoothed probability plots. The chapter then investigated tail behaviour and eventually made assessments of the least and most volatile months on the various exchanges through resampling leading that led to the average monthly volatility ranking plots.

Heteroscedastic models generally used in the finance industry for describing the evolution of risk are the classical GARCH models. There is a multiplicity of these models, each designed to capture some stylized fact of return series. In line with our research questions, we have extended the analysis to include switching models which evolve according to given states of the markets. Each of these states, described as high or low volatility regimes, should have its own unique statistical properties.

Analysis

The data for the analyses are a sample of the daily closing index levels from the equity markets of Botswana, Ghana, Kenya and Nigeria spanning January 04, 2011 to December 31, 2017 giving 1540 data points. The indices for each of these exchanges are a composite of the price-weighted average of all shares listed and they are a proxy for the overall performation of the various exchanges.

In Table 6, we provide statistical summaries of the index levels of the market data for the sample period (All the tables, figures and results were produced in Python and R code reproduced in Appendices A and B).

Table 6: Summary of level of indices

	GSE	BSI	KSE	NSE
count	1549	1549	1549	1549
mean	1710.25	122.89	167.20	8737.27
std	494.23	31.52	51.81	1292.12
min	940.04	65.14	78.16	6411.47
25%	1142.61	92.36	130.40	7320.10
50%	1821.66	132.06	158.74	8992.35
75%	2156.68	146.43	213.19	9679.25
max	2439.20	177.51	265.78	11096.92

Source: Author's computations, (2019)

A plot of the index levels for the respective exchanges as it evolves over our sample period is shown in **Figures 1-4**.

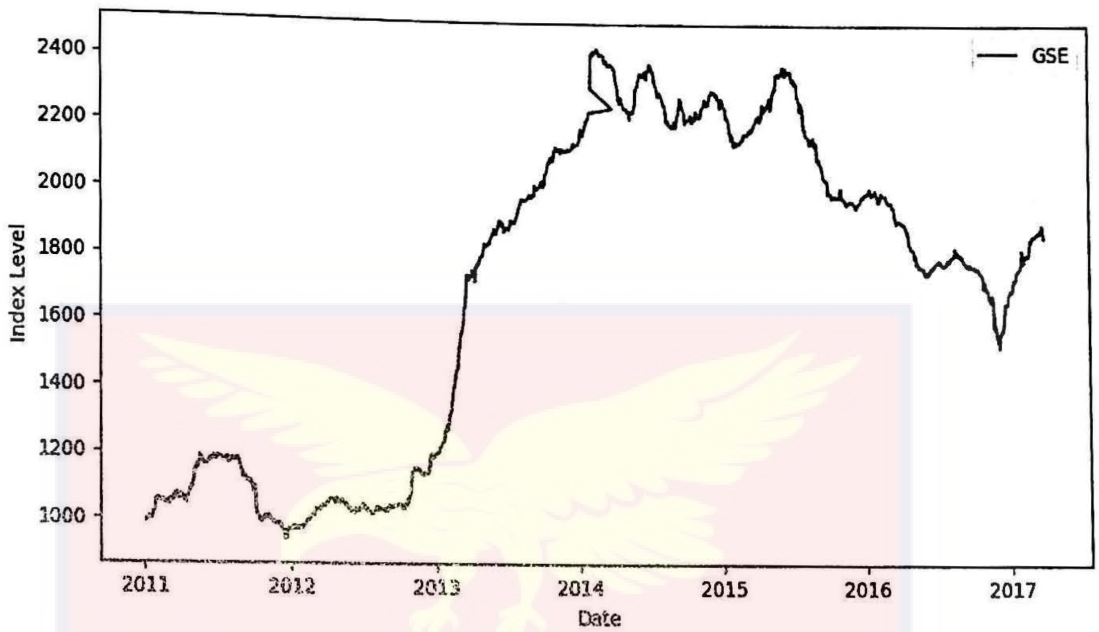

Figure 1: Index levels of GSE over sample period
Source: Korkpoe (2019)

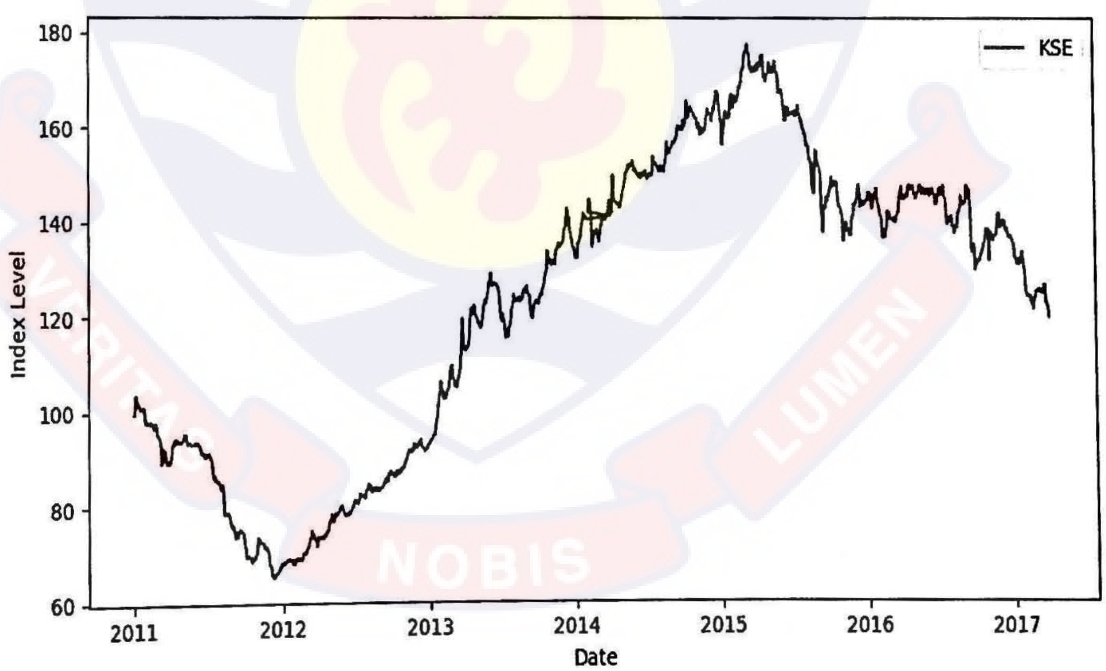

Figure 2: Index levels of KSE over sample period
Source: Korkpoe (2019)

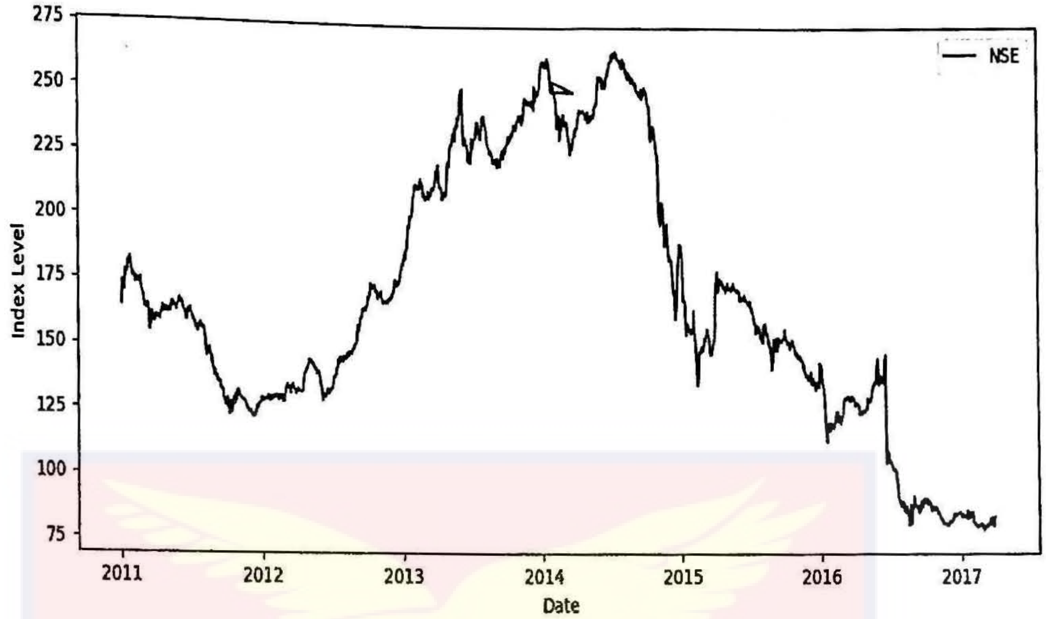

Figure 3: Index levels of NSE over sample period
Source: Korkpoe (2019)

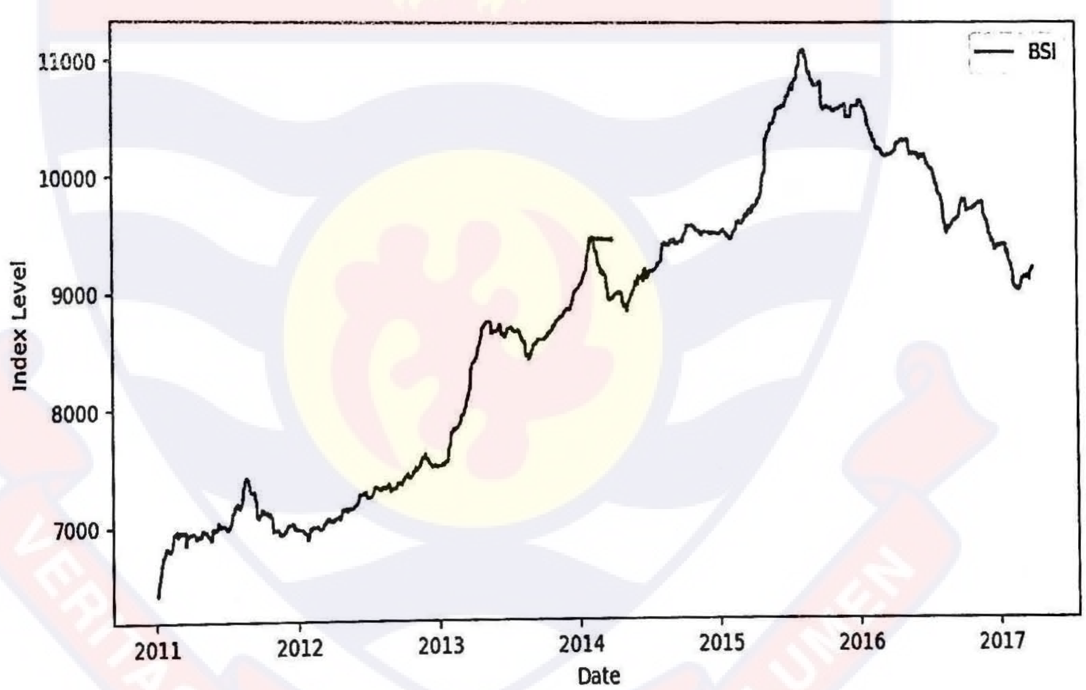

Figure 4: Index levels of BSI over sample period
Source: Korkpoe (2019)

The indices do not exhibit the typical consistently upward trends of market indices of the developed and emerging markets (Cont, 2001). The GSE after an initial rise to the middle of 2011, tumbled and stayed almost flat till the beginning of 2013 when it rose consistently eventually peaking in the first quarter of 2014. The market showed marked swings thereafter touching to a low point at the end of 2016. From 2017, there was an uptick in the level and this trend continued till the end of the sample period. The KSE showed similar characteristics. The market index fell for the whole of 2011 but recovered for a market rally stretching from the start of 2012 till the middle of 2015. The index then tumbled and oscillated around a downward trend till the end of 2017.

The NSE index levels exhibited much turbulence during the sample period. Overall, it tumbled for the whole of 2011 recovering only slightly mid-year. The market recovered thereafter and showed an upward trend falling on occasions till mid-2014 when it resumed the downward march, reaching a low in the first quarter of 2015. It bottomed out thereafter only slightly during the year eventually falling sharply mid-year and remained in that state till the end of the year. The BSI showed remarkable resilience rising gradually with what seems to be occasional corrections till about the third quarter of 2015. From there, it tumbled till the end of the year. Again, looking at Figures 1-4, the various bourses fell largely from the middle of 2015 to the end of the sample period.

The log-returns of the index levels for the composite indices were computed from $r_t = \log\left(\frac{P_t}{P_{t-1}}\right)$, where P_t is taken to be a proxy for the index level at a time t. A statistical summary is provided in Table 7.

Table 7: Summary of the log-returns of indices

	GSE	KSE	NSE	BSI
Count	1548	1548	1548	1548
Mean	0.0004	0.000118	-0.00044	0.000233
Std	0.005367	0.007085	0.014169	0.002453
Min	-0.02758	-0.05141	-0.26436	-0.01896
25%	-0.00178	-0.00333	-0.00553	-0.00041
50%	0.000151	0.000206	-0.0003	0.00009
75%	0.002341	0.003825	0.005372	0.000916
Max	0.027212	0.040281	0.081686	0.019947
Skew	0.320365	-0.538311	-4.341482	0.039821
Kurtosis	5.19146049	6.61721024	80.89740691	13.27760729

All the returns of the indices as shown in Table 7 exhibit deviations from normality stemming mainly from fat-tails. The skewness is not too severe although that of NSE is slightly elevated. The skew of GSE and BSI is slightly to the right, a departure from the observations of the stylised properties characterising equity market returns (Cont, 2001). The kurtosis of NSE is noteworthy. This will make investors wary of investing in this market as there is an elevated risk to the left of the tails. A graph of the distribution of the returns is shown in Figures **5-8**.

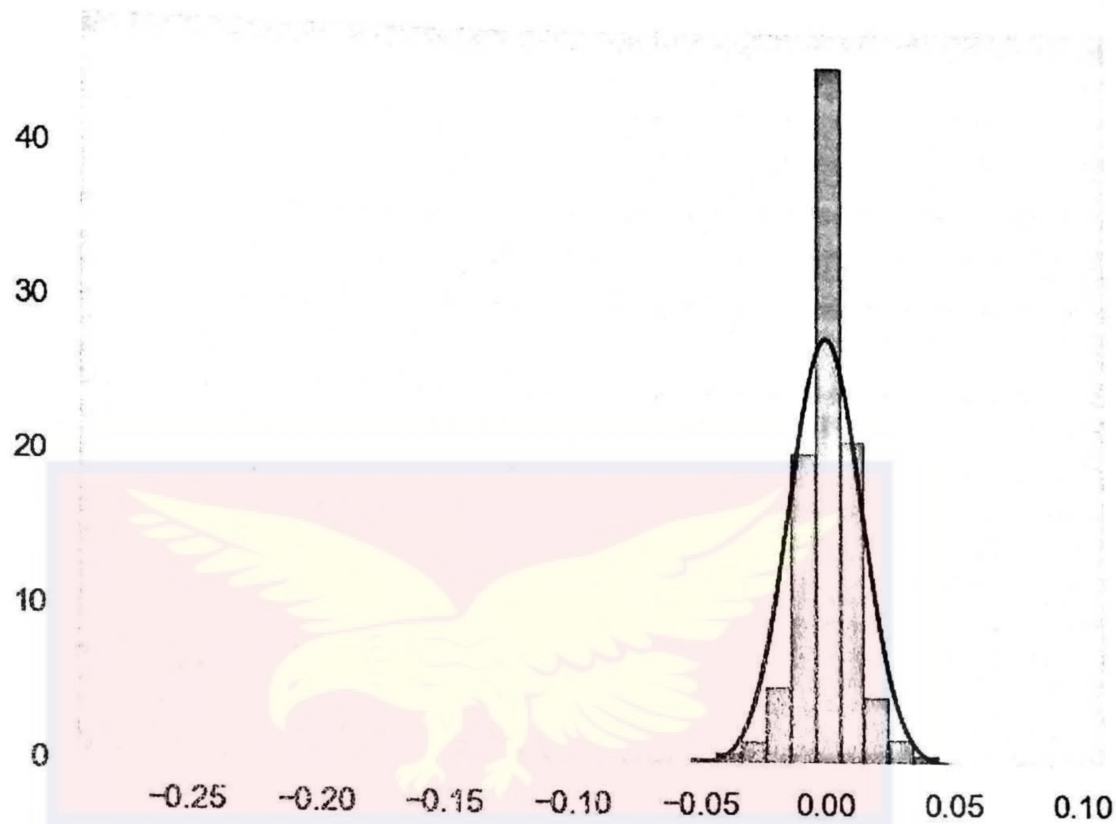

Figure 7: Distribution of NSE Index Returns

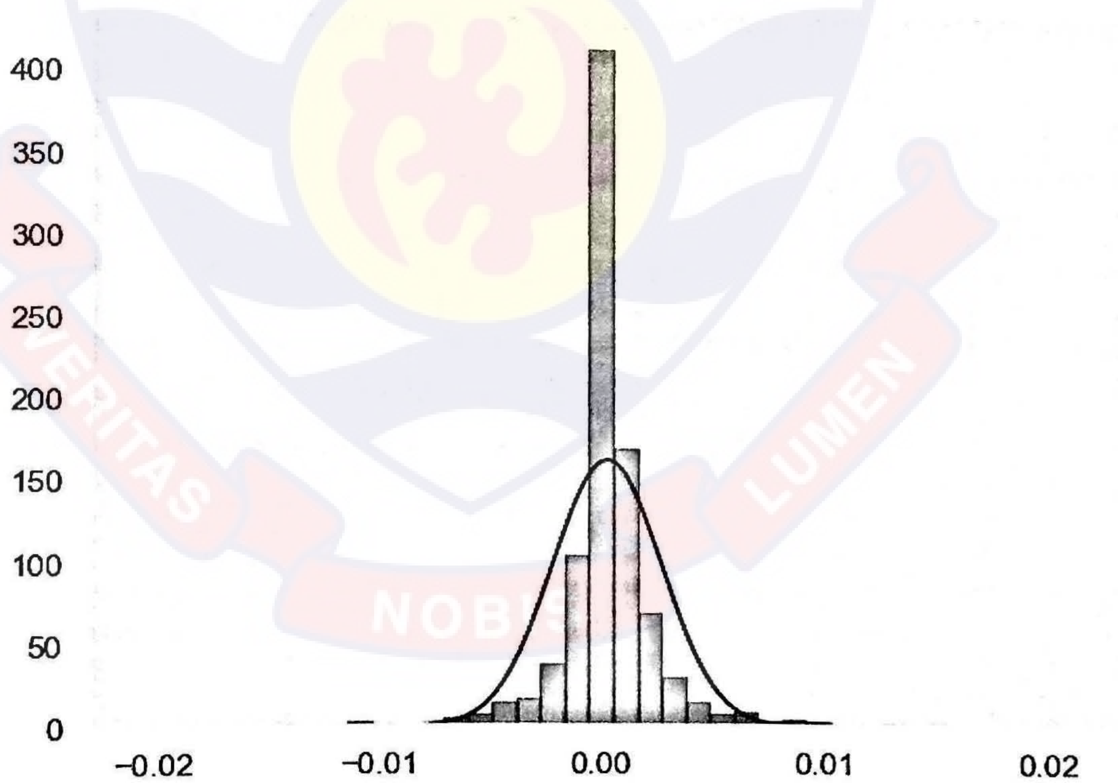

Figure 8: Distribution of BSI Indices Returns

We plot the time series of the return as shown in **Figures 9-12**.

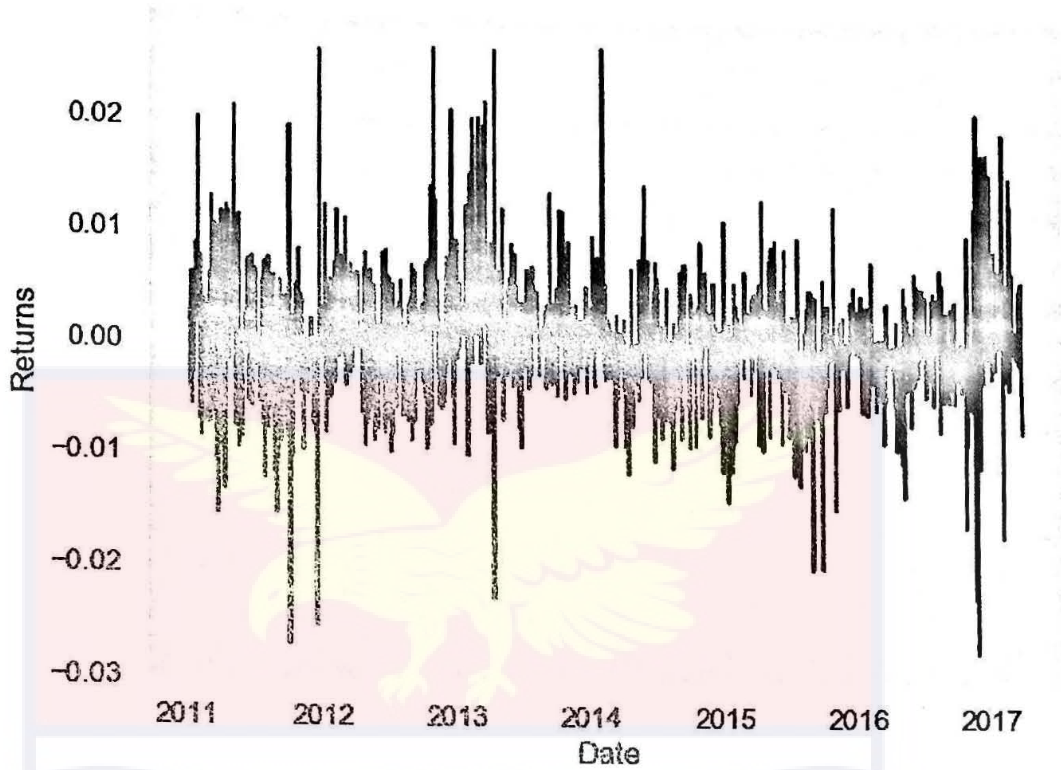

Figure 9: Time Series of GSE Composite Index Returns

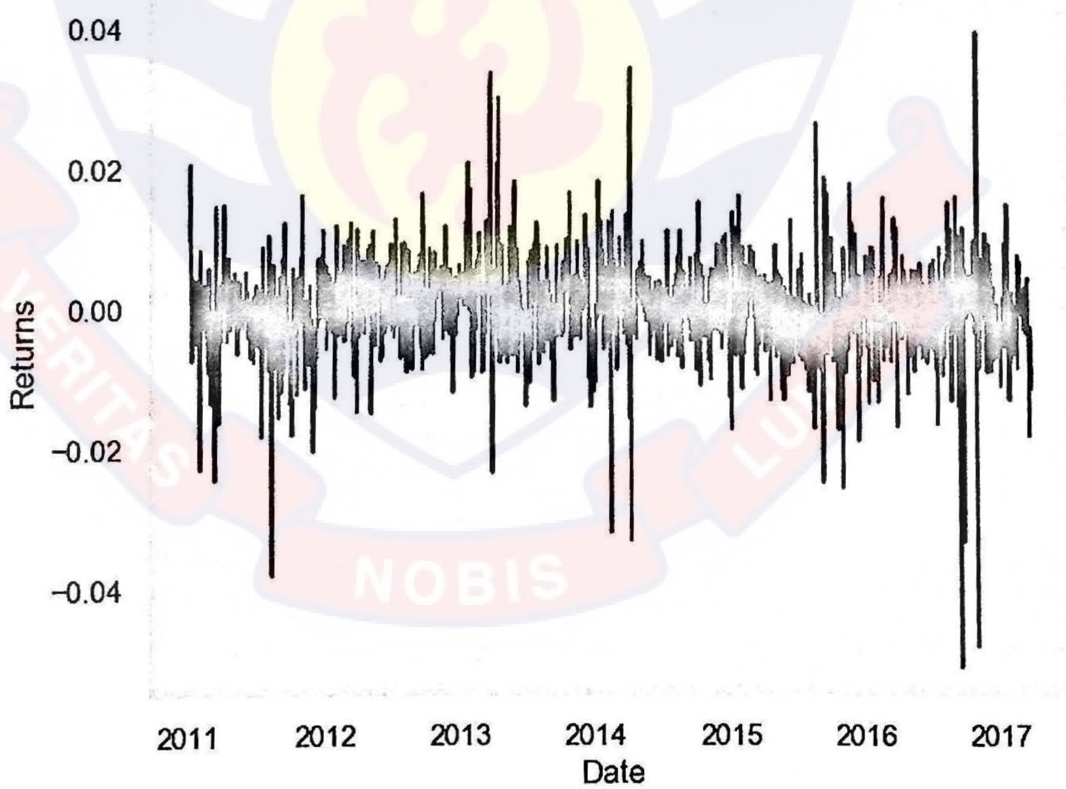

Figure 10: Time Series of KSE Composite Index Returns

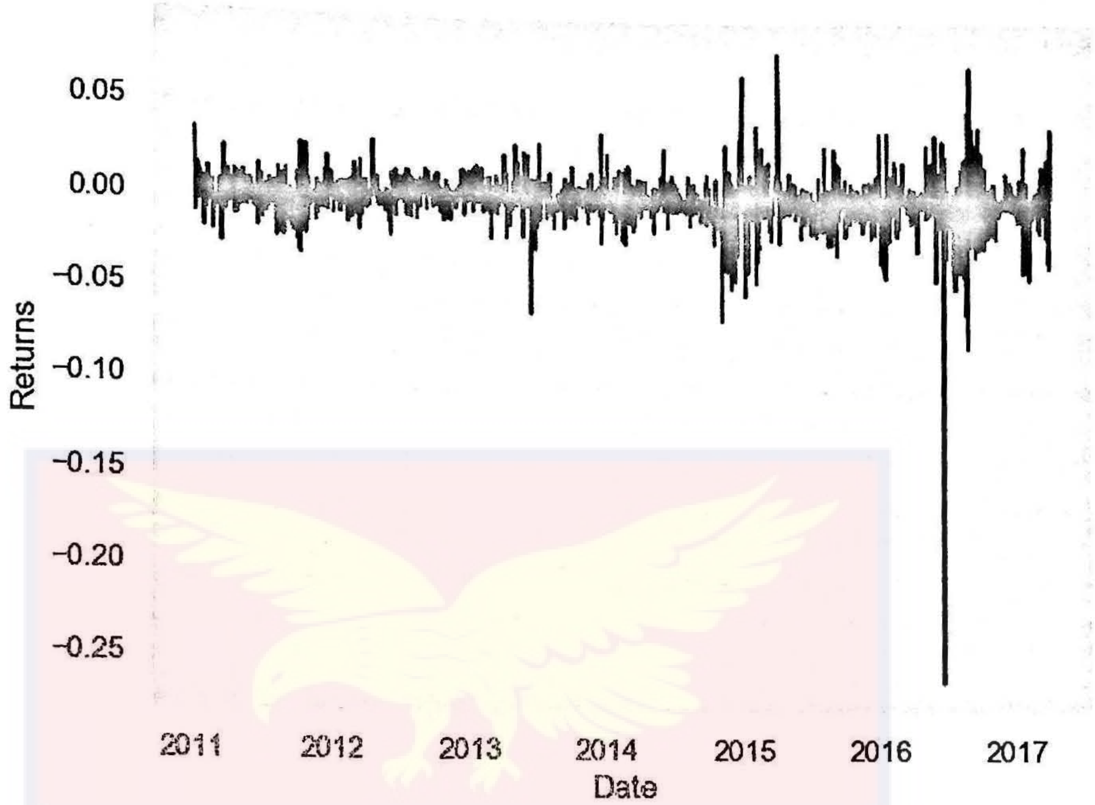

Figure 11: Time Series of NSE Composite Index Returns

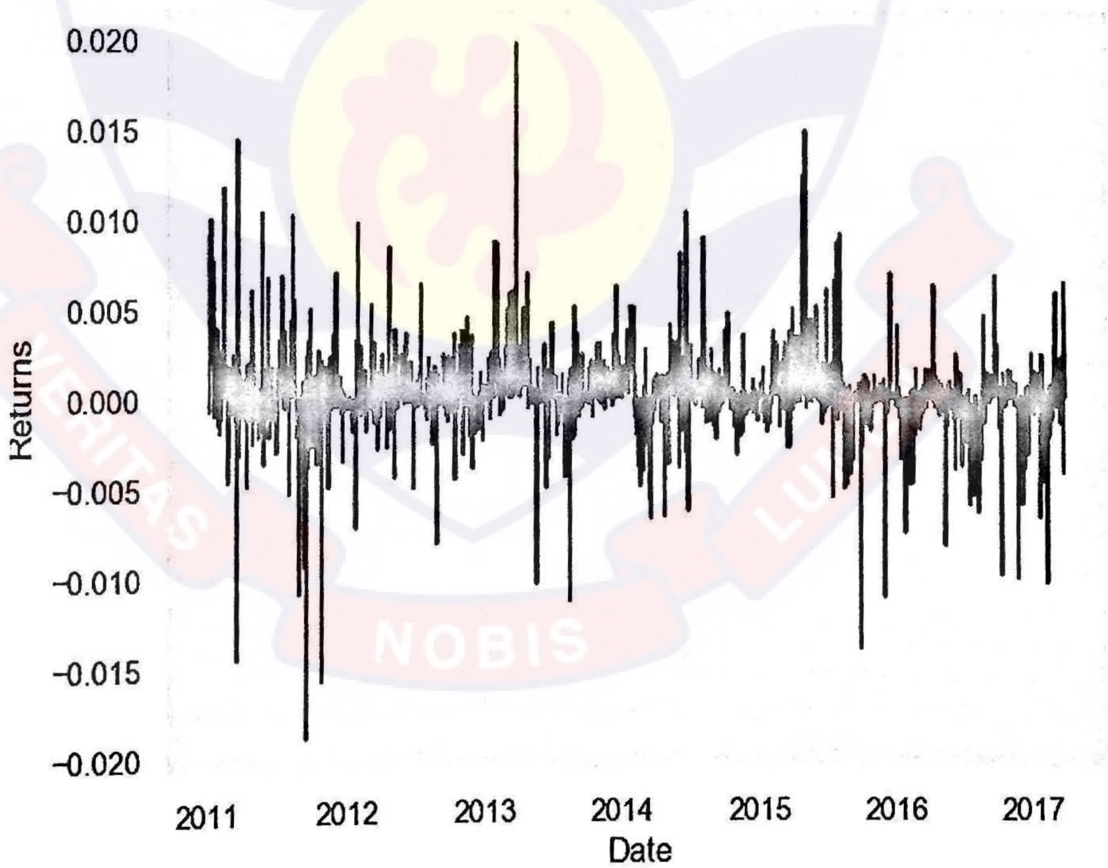

Figure 12: Time Series of BSI Composite Index Returns

Returns for most part of the sample period have been volatile. The GSE, KSE and BSI have been choppy usually at the beginning and ending of the year. The NSE has been relatively quiet for much of the sample period except in mid-2016 when the returns drop sharply signifying some material information reaching the market. Volatility clustering is much pronounced for GSE, KSE and BSI. KSE saw some sharp declines in the last quarter of 2016. Similar observations can be made for BSI at the end of 2011. Market volatility has been much more intense for the Ghanaian bourse.

Establishing Market Regimes

Evidence of market regime switching was established using the **bcp** package of Erdman and Emersion (2007).

Ghana

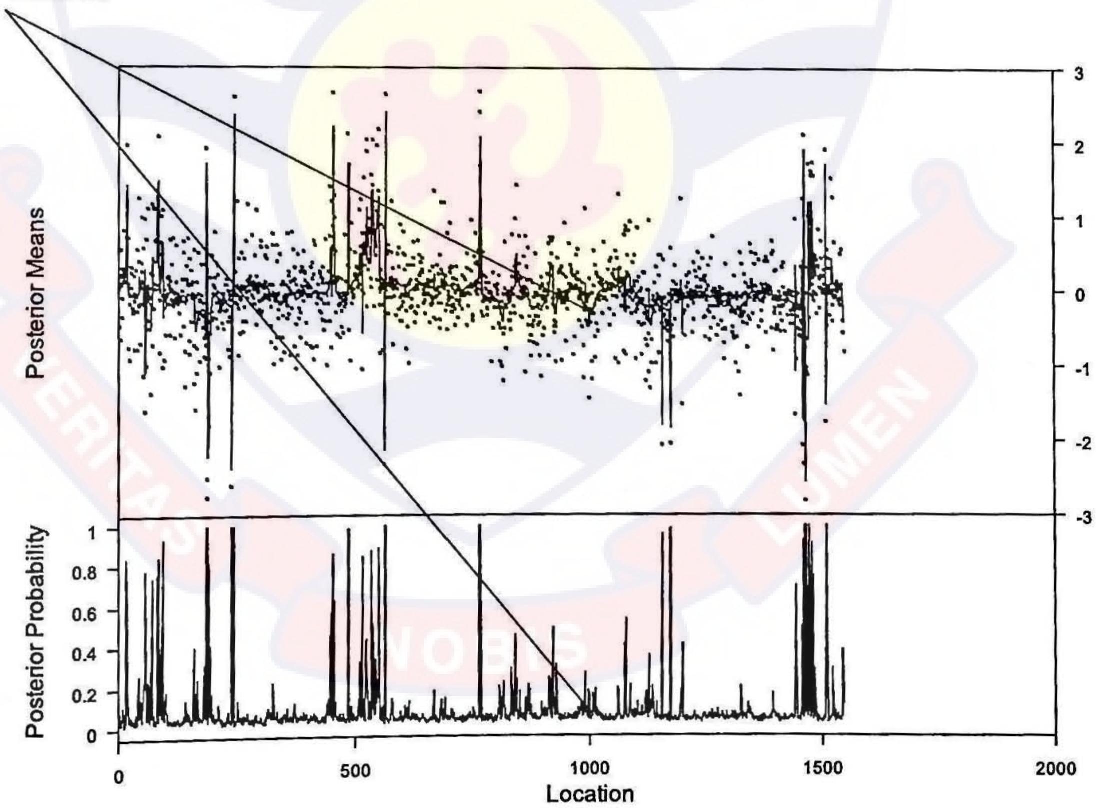

Figure 13: Regime changes in GSE returns for the sample period

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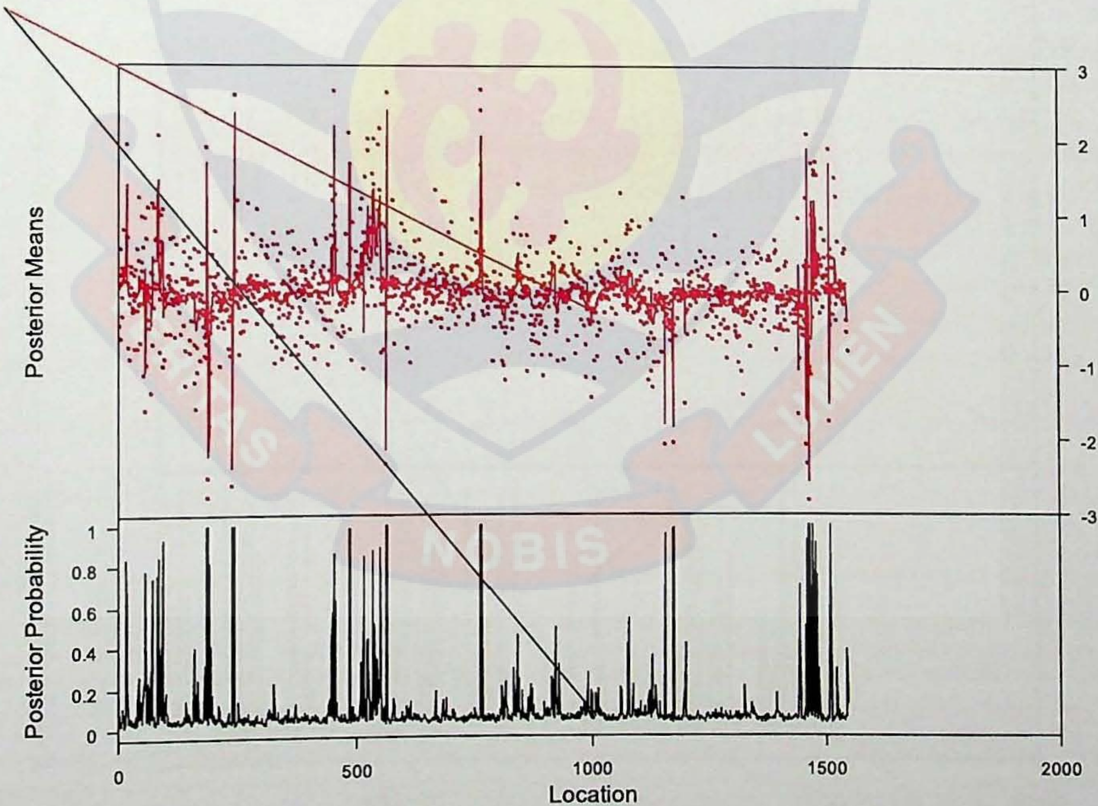

Figure 13: Regime changes in GSE returns for the sample period

Figure 13 shows the nature of the change points in the returns of the GSE. The GSE indeed shows clear market regime changes where the low volatility regimes are interspersed with high volatility regimes sometimes briefly. There is a dominance of low regimes during the sample period. The market seems to rally quickly following the uptick in volatility.

Regime Changes in Nairobi Stock Exchange

Stocks of the KSE have exhibited more volatility over the sample period as show in **Figure 14**. There are frequent regime changes in the returns with the high volatility more than dominate the market outcomes.

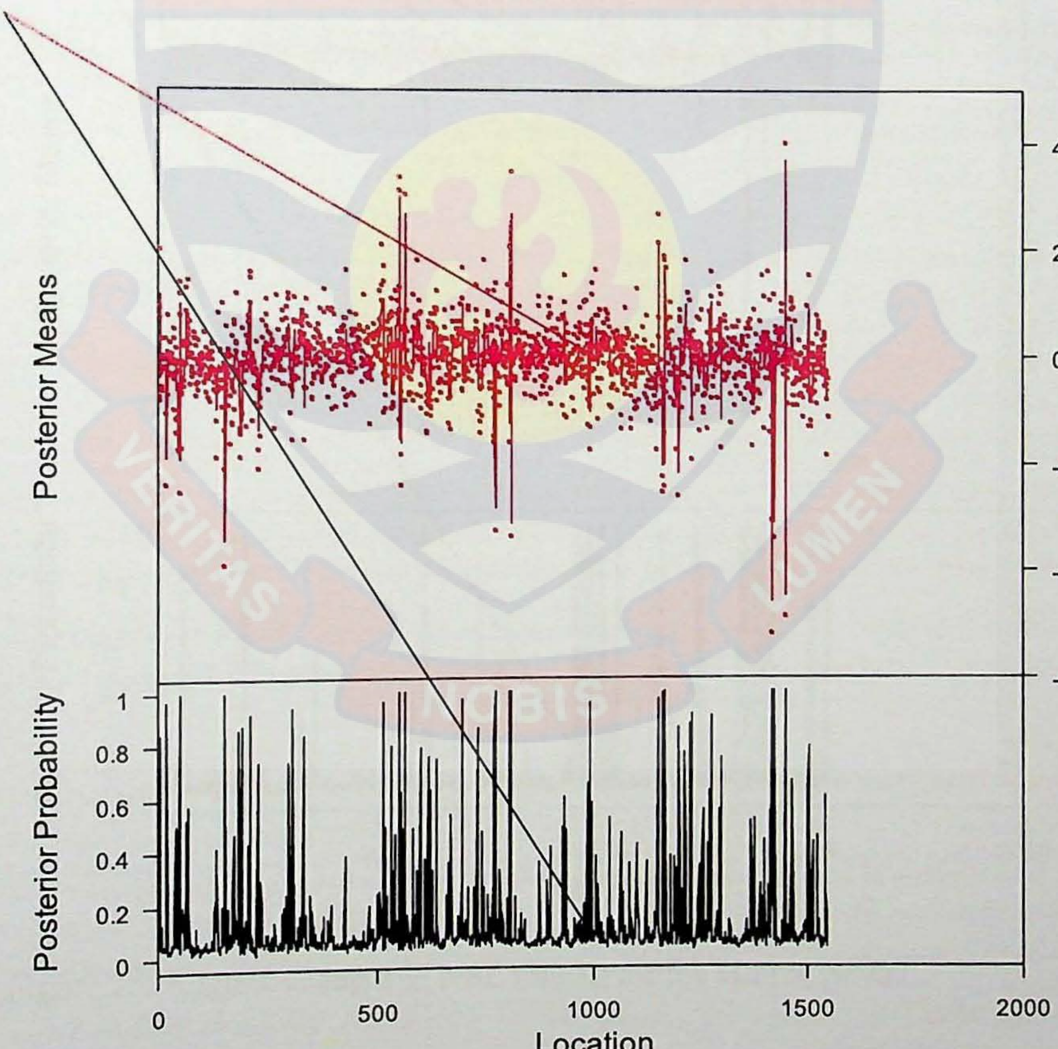

Figure 14: Regime changes in KSE returns for the sample period

Regime Changes in Nigeria Stock Exchange

The NSE has been relatively quiet at the beginning of the sample period and only assuming a more volatile nature from the the second half of the period. In fact, the second half has seen very turbulent market outcomes with the high volatility regimes dominating market outcomes. This is clearly shown in **Figure 15**. The market tanked briefly in the middle of 2016 before recovering.

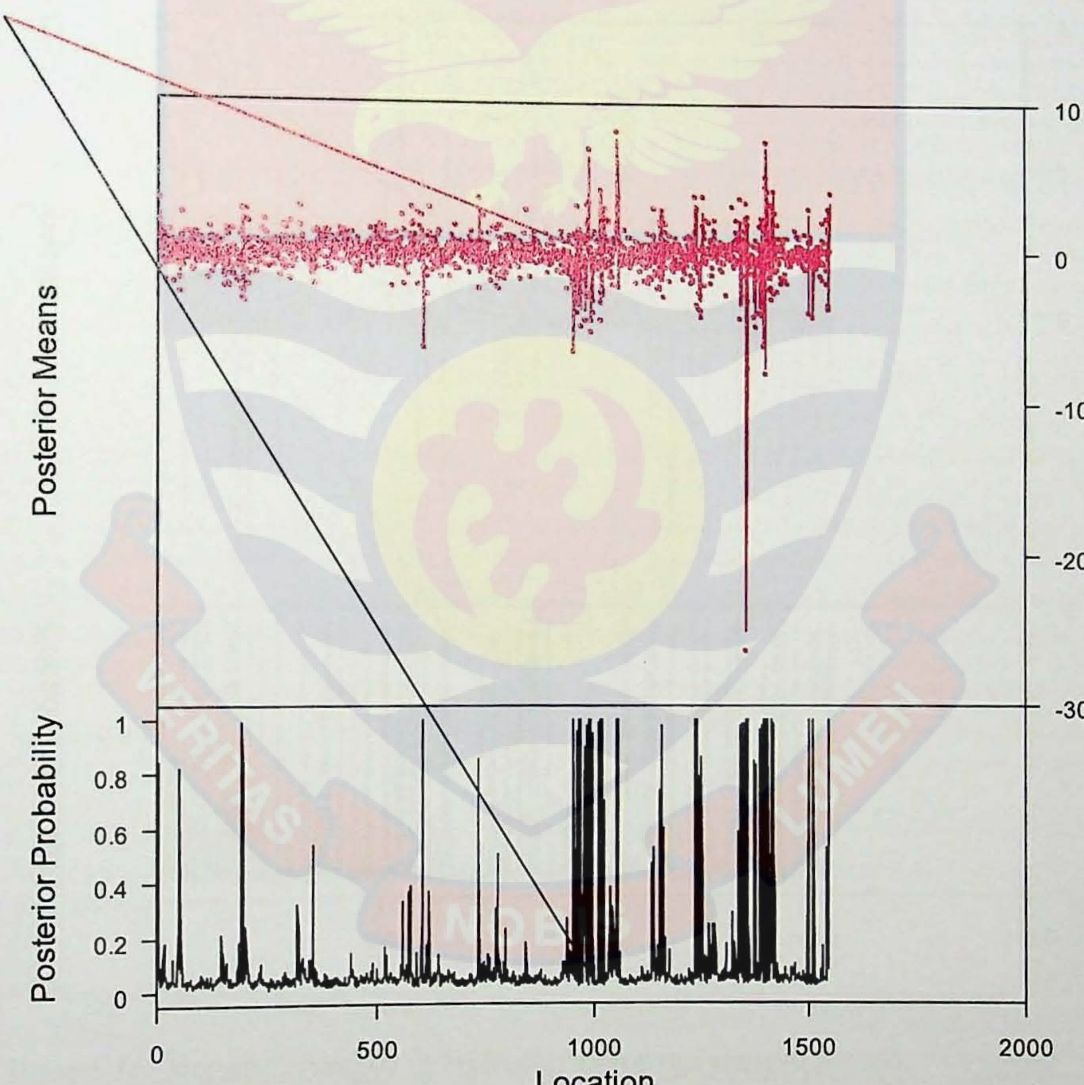

Figure 15: Regime changes in NSE returns for the sample period

Regime Changes in the Botswana

There is no pattern to the returns in the BSI as shown in **Figure 16**. The market has been most volatile with high volatility regime dominating across in the sample. Episodes of low volatility have been rather brief with market yielding to the high volatility regimes. The level of volatility does not compare to any of the markets in the sample.

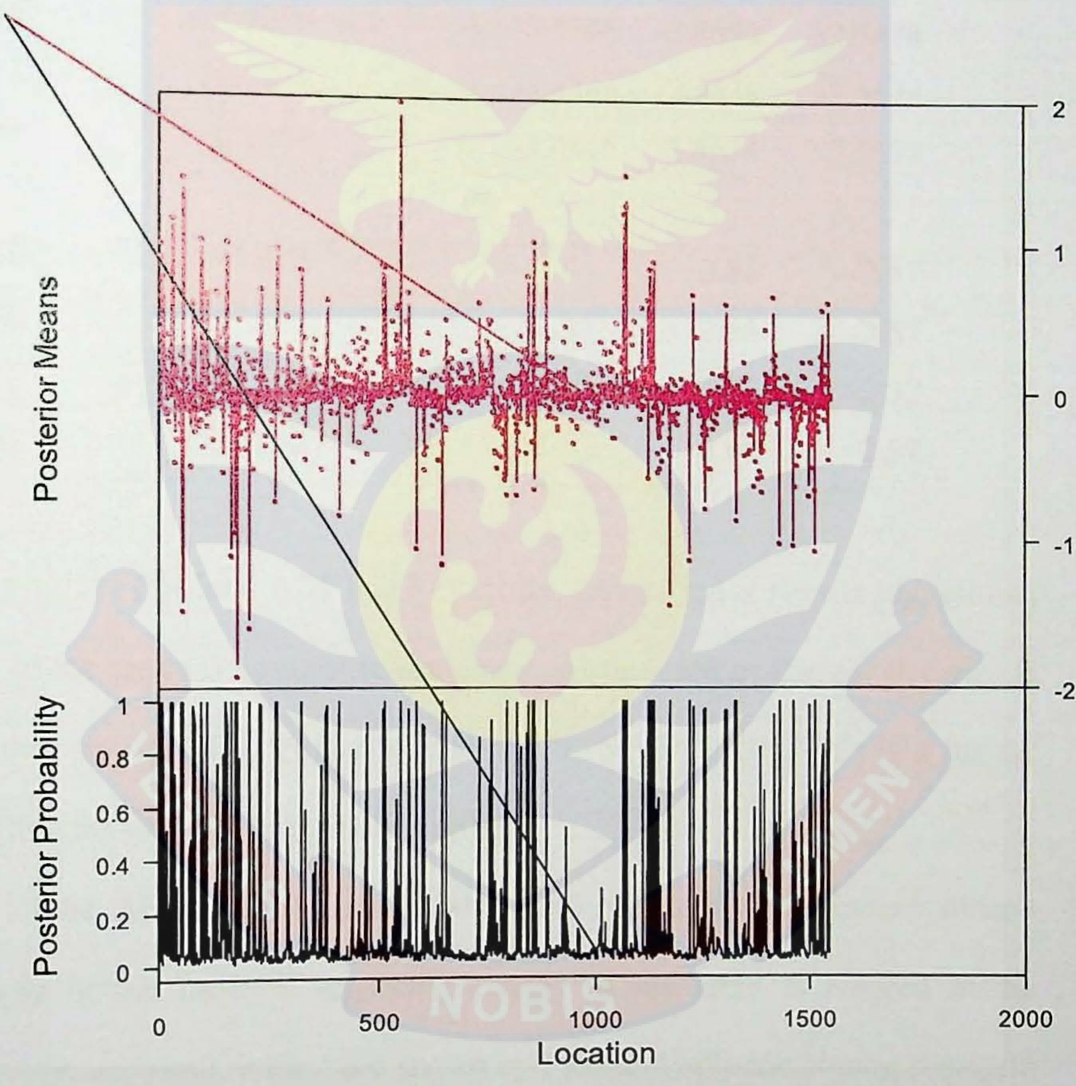

Figure 16: Regime changes in BSI returns for the sample period

Tests for Stationarity

The heteroscedasticity models require the data to be covariance stationary; hence we conducted formal unit root tests of stationarity using the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests. The null of the ADF and PP is that the series is non-stationary and the alternative which is one-sided says it is stationary. The results are displayed in Table 8.

Table 8: ADF and PP tests of stationarity results

Index	Test statistic	p-value	Critical values 1%	5%	10%
		ADF			
GSE	-8.5341	0	-3.4346	-2.8634	-2.5678
KSE	-27.8239	0	-3.4346	-2.8634	-2.5678
NSE	-18.4741	0	-3.4346	-2.8634	-2.5678
BSI	-10.8264	0	-3.4346	-2.8634	-2.5678
		PP			
GSE	-41.672	0	-3.43	-2.86	-2.57
KSE	-27.827	0	-3.43	-2.86	-2.57
NSE	-35.368	0	-3.43	-2.86	-2.57
BSI	-38.833	0	-3.43	-2.86	-2.57

The p-values (for both ADF and PP) for the various index returns suggest that each of the series is covariance stationary. Again, each of the test statistics is less than the specified critical values of 1%, 5% and 10%. This is a further confirmation of a stationary return series.

The ADF and PP tests however are silent on the presence of any changes in the DGP. If any such change in the DGP is ignored in the stationary test result, it has been shown by Perron (1989) and Nunes, Newbold and Kuan (1997) that the power of the stationary tests is reduced considerably. Further evidence of this problem in stationary tests on return series not accounting for breaks in the DGP is provided in Ben-David and Papell (1995) and Darné and Diebolt (2004). To address this problem, Zivot and Andrews (2002) pioneered a unit root test which assumes a latent change point in the

DGP. The proposed test has as its null and alternate hypotheses respectively as follows:

H_0: The data generation process contains a unit root with no break in the series and H_A: The data generation process is trend and breaks stationary.

We conducted a Zivot-Andrews (ZW) test to see if there was evidence of breaks in the stationary series. The results are shown in Table 9.

Table 9: Result of the Zivot-Andrews test

			Critical values		
Index	**Test statistic**	**p-value**	**1%**	**5%**	**10%**
GSE	-9.227	0.00	-5.28	-4.81	-4.57
KSE	-28.444	0.00	-5.28	-4.81	-4.57
NSE	-18.793	0.00	-5.28	-4.81	-4.57
BSI	-11.461	0.00	-5.28	-4.81	-4.57

The p-values provide evidence to reject the null hypothesis in favour of the alternative that there are breaks in the DGP. This is affirmed by the test statistics being far less than the corresponding critical values at 1%, 5% and 10% levels.

Test for GARCH Effects

We tested the univariate series for ARCH effects using both the Ljung-Box (Ljung & Box, 1978) and Box-Pierce (Box & Pierce, 1970) tests. We followed recommendations of Hyndman and Athanasopoulos (2018) in choosing 10 lags based on a lack of seasonality in each of the return series. The results of the Ljung-Box and Box-Pierce test statistics and the respective p-values are shown in Table 10.

Table 10: Result of the ARCH tests

	Ljung-Box	p-value	Box-Pierce	p-value
GSE	305.5834	0	304.1743	0
KSE	201.9597	0	201.5328	0
NSE	34.4425	0.0002	34.3465	0.00016
BSI	262.6499	0	261.6524	0

The test statistics have a χ^2_{m-K}-distributions with *(m-K)* degrees of freedom, where m is the lag length and K is the number of parameters estimated from the model. Large values of the test statistics suggest that the distributions do not emanate from a typical white noise process. As the results show, the p-values for each of the test both for the Ljung-Box and Box-Pierce are below 5%. This suggests that the residuals have ARCH effects.

Model Estimation

We run a total of 144 GARCH models on the demeaned return series employing both single and regime-switching models with various conditional error distributions. These error distributions span the normal, Student-*t* to the Generalised Error Distributions (GED) and the skewed versions. We relied on Hansen and Lunde (2005) to run the tests limited to GARCH (1, 1) models with various specifications for tail behaviour. The first models were single regime and then followed by two regime processes. The Deviance Information Criteria (DIC) was compared for each of the models and used as basis for model fit (Spiegelhalter, Best, Carlin, & Van Der Linde, 2002). Gelman et al. (2013) discuss extensively other model choice criteria like Akaike Information Criteria (AIC), Bayesian Information Criteria (BIC) and Watanabe Akaike

Information Criteia (WAIC). They noted that AIC and BIC rely on point estimates from the maximum likelihood estimate whereas WAIC, with its desirable Bayesian statistical and theoretical properties, falls short in dealing with time series data. The DIC is, in their view, much quicker and easier in Monte Carlo approximation.

Single Regime Models

We run 72 single-regime models using the MSGARCH package (Ardia, Bluteau, Boudt, Catania, & Trottier, 2019) of the open-source R statistical environment (R Core Team, 2019). The models covered the normal, E-GARCH and GJR-GARCH with their respective tail specifications of normal, skewed normal, student-*t*, skewed student-*t*, generalised error and skewed generalised error distributions. For each model, 15,000 iterations were chosen, each consisting of 5,000 burn-in to start the Markov Chain Monte Carlo (MCMC) process. Out of the 10,000 left, we sampled at every tenth to give us a chain length of 1000 as the posterior. The results are shown in **Table 11**.

Table 11: Single-regime DICs of the GARCH models with conditional tail distributions

		Conditional Distributions					
		Normal	Skewed Normal	Student-t	Skewed Student-t	GED	sGED
GARCH (1, 1)	GSE	2425.7176	2428.4456	1926.2971	1924.5499	1869.1678	**1842.8228**
	KSE	3061.3383	3049.1730	2885.9408	**2885.5421**	2911.6457	2913.1322
	NSE	4818.5731	4816.3657	**4453.8343**	4453.9911	4494.3724	4495.6045
	BSI	483.3397	463.4041	-1045.0873	-1054.8554	-1063.200	**-1157.875**
EGARCH (1, 1)	GSE	2435.2398	2436.4564	1918.2585	1936.6205	1867.3719	**1844.2859**
	KSE	3072.9794	3059.1691	**2894.3320**	2910.7338	2918.4230	2917.6803
	NSE	4813.8416	4811.3648	**4463.9029**	4516.7100	4502.7242	4556.3219
	BSI	365.3243	356.1232	-1056.4367	-1052.7932	**-1078.570**	84.1033
GJR-GARCH (1, 1)	GSE	2429.1157	2430.6069	1931.6077	1961.2549	1873.3336	**1860.4221**
	KSE	3064.3416	3051.7532	**2887.5237**	2895.9578	2916.2160	2919.2124
	NSE	4822.5142	4818.7720	4454.2835	**4453.3486**	4497.2398	4496.5685
	BSI	446.7274	430.2870	-1039.3239	-1029.2901	-1061.655	**-1144.322**

Evidence of the fat-tailed distributions is present in the results. In all the models, heavy-tailed distributions provide a better fit to the data. Fat-tails are a characteristic seen in the emerging and frontier markets (Cerović Smolović, Lipovina-Božović, & Vujošević, 2017; Assaf, 2015). Much of the studies in equity returns in the Middle East and North African regions exhibit these characteristics (Aloui & ben Hamida, 2014). This is highlighted in Table 7. The GSE, KSE and BSI showed particularly heavy-tailed distributions shown earlier in Figures 5, 6 and 8.

Remark 1

The code snippet to run the analysis in Table 11 is reproduced in Appendix B.

Two-regime Models

We run two regime-switching models on the same data and models to enable us compare their performances and fits for the data. The DICs for the corresponding two-regime GARCH models is shown in Table 12 with the minimum values of the DICs highlighted.

Table 12: Two-regime DICs of the GARCH models with conditional tail distributions

		Conditional Distributions					
		Normal	Skewed Normal	Student-t	Skewed Student-t	GED	sGED
GARCH (1, 1)	GSE	1986.7152	2051.0894	1876.9550	1847.2593	1864.0120	**1840.5587**
	KSE	2912.4524	2906.9608	2887.7502	2889.7537	**2883.5687**	2885.9944
	NSE	4523.7139	4527.0571	**4452.2951**	4460.0540	4468.4455	4473.7267
	BSI	-909.2936	-949.1059	-1145.8760	**-1241.9889**	-1123.1246	-1228.1422
EGARCH (1, 1)	GSE	2032.1324	2038.9862	1972.6570	1947.0340	1879.0032	**1838.9819**
	KSE	2918.0721	2917.7603	2912.1950	2897.1373	**2894.2006**	2915.8780
	NSE	4507.3520	4505.5317	**4454.9198**	4462.2981	4468.7972	4470.3177
	BSI	-928.6000	-958.7706	-1096.0921	-1143.7046	-1112.5417	**-1211.6356**
GJR-GARCH (1, 1)	GSE	1981.2280	2012.7841	1890.7492	1901.2833	1872.6219	**1841.4586**
	KSE	2909.7519	2905.6261	2891.8240	2891.4228	**2884.8118**	2911.8238
	NSE	4520.2918	4510.3730	4441.8089	**4433.4383**	4474.3290	4481.3943
	BSI	-906.6659	-939.3307	-1112.4444	-1213.1226	-1118.0404	**-1222.7124**

Table 12 shows the models with the minimum DICs highlighted. Compared to Table 11, the DICs for the highlighted models in Table 12 provide a better fit for the data. Again, the effects of fat-tails in the extreme of the distributions are present in the two-state regimes as well.

Remark 2

The code snippet to run the analysis in Table 12 is reproduced in Appendix B. The models with the minimum DICs in the class of models in Table 12 were extracted and shown in Table 13 for further analysis.

Table 13: Models with the minimal DICs extracted

Index	GARCH Model	Conditional Distribution	DIC
GSE	EGARCH (1,1)	skewed GED	**1837.5056**
KSE	GARCH (1, 1)	GED	**2883.5687**
NSE	GJR-GARCH (1, 1)	skewed Student-*t*	**4433.4383**
BSI	GARCH (1, 1)	skewed Student-*t*	**-1241.9889**

Subsequently, we showed the GARCH estimates for the various indices in Tables 14, 15, 16 and 17.

Table 14: GARCH estimates for the GSE All Share Index

	Mean	SD	SE	TSSE	RNE
α_{01}	0.0809	0.0224	0.0007	0.0021	0.1172
α_{11}	0.153	0.0382	0.0012	0.0033	0.136
α_{21}	0.0283	0.0341	0.0011	0.0033	0.1072
β_1	0.4912	0.1036	0.0033	0.0099	0.1101
ν_1	0.7957	0.0357	0.0011	0.0024	0.2127
ξ_1	1.0649	0.0121	0.0004	0.0007	0.3054
α_{02}	1.138	0.6163	0.0195	0.0538	0.1312
α_{12}	0.1519	0.0809	0.0026	0.0066	0.1517
α_{22}	0.0001	0	0	0	0.0939
β_2	0.3753	0.1819	0.0058	0.0134	0.1835
ν_2	1.0501	0.4079	0.0129	0.0406	0.1012
ξ_2	0.7758	0.1302	0.0041	0.0108	0.1455
P_{11}	0.9915	0.0051	0.0002	0.0004	0.1959
P_{21}	0.0801	0.0423	0.0013	0.0024	0.3032

Table 15: GARCH estimates for the KSE All Share Index

	Mean	SD	SE	TSSE	RNE
α_{01}	0.138	0.0306	0.001	0.0013	0.5869
α_{11}	0.4219	0.0637	0.002	0.003	0.4635
β_1	0.2192	0.0634	0.002	0.0035	0.3344
ν_1	1.5491	0.1213	0.0038	0.0056	0.4696
α_{02}	0.8952	0.2502	0.0079	0.0164	0.2333
α_{12}	0.0541	0.0411	0.0013	0.0017	0.5536
β_2	0.0409	0.082	0.0026	0.0057	0.2054
ν_2	1.0363	0.1901	0.006	0.01	0.364
P_{11}	0.8627	0.0791	0.0025	0.0043	0.3369
P_{21}	0.904	0.1436	0.0045	0.0091	0.2479

Table 16: GARCH estimates for the NSE All Share Index

	Mean	SD	SE	TSSE	RNE
α_{01}	0.3264	0.6508	0.0206	0.0328	0.3943
α_{11}	0.3456	0.0854	0.0027	0.0051	0.2838
α_{21}	0.021	0.0228	0.0007	0.0013	0.3192
β_1	0.5191	0.0926	0.0029	0.007	0.1729
ν_1	4.631	8.6814	0.2745	0.4835	0.3224
ξ_1	0.9659	0.0781	0.0025	0.0032	0.5836
α_{02}	5.1929	5.9048	0.1867	0.8106	0.0531
α_{12}	0.8099	0.1676	0.0053	0.0111	0.2289
α_{22}	0.0018	0.0054	0.0002	0.0005	0.1409
β_2	0.0113	0.0661	0.0021	0.0036	0.341
ν_2	55.4033	25.6674	0.8117	2.3201	0.1224
ξ_2	0.9862	0.3944	0.0125	0.0582	0.0459
P_{11}	0.9933	0.0179	0.0006	0.0007	0.6664
P_{21}	0.1194	0.1576	0.005	0.0209	0.057

Table 17: GARCH estimates for the BSI All Share Index

	Mean	SD	SE	TSSE	RNE
α_{01}	0.0005	0.0002	0.0000	0.0000	0.0939
α_{11}	0.0013	0.0020	0.0001	0.0002	0.1147
β_1	0.3954	0.1570	0.0050	0.0166	0.0900
ν_1	96.5569	3.9439	0.1247	0.4185	0.0888
ξ_1	2.9389	2.5134	0.0795	0.6672	0.0142
α_{02}	0.3295	0.0409	0.0013	0.0030	0.1799
α_{12}	0.7090	0.1204	0.0038	0.0111	0.1185
β_2	0.0002	0.0001	0.0000	0.0000	0.1519
ν_2	2.1069	0.0082	0.0003	0.0009	0.0884
ξ_2	1.0488	0.0238	0.0008	0.0016	0.2273
P_{11}	0.4597	0.0751	0.0024	0.0061	0.1526
P_{21}	0.1969	0.0285	0.0009	0.0019	0.2308

The issue of convergence in Bayesian methods remains an open and sometimes contentious among researchers. A survey of the methods from various authors can be found in Cowles and Carlin (1996). We adopt the criteria in Geweke (1992) using the relative numerical efficiency (RNE) in assessing convergence. The RNE is given by $(\frac{SE}{TSSE})^2$ where SE is naive standard error of the mean assuming the autocorrelation in the MCMC chain is ignored and TSSE is the time series standard error that is based on the estimate of the spectral density at zero. Geweke (1992) puts the convergence rates at between 0.1 and 1.0. RNE values typically less than 0.01 point to serial correlation in the parameter in the MCMC algorithm (Geweke & Keane, 2001). Looking at Tables 13 to 17, we see that the RNE values meet our criteria.

We checked for the acceptance rate for the MCMC sample of the Metropolis-Hasting sampler which is given by the ratio of the accepted samples to the Markov chain length. It asseses how efficient the proposal distribution approximates the true distribution of the chain. Chib and Greenberg (1995) suggest value of acceptance rate around 23%-45%. This was taken from the recommendation of Muller (1993) who recommended value of around 50% for the random walk used in this work. The acceptance rates for the GSE, KSE, NSE and BSI are respectively, 28.1%, 27.3%, 27.3% and 27.4%.

Backtesting the Models

We backtest the models with Engle and Manganelli's (2004) DQ test statistic specified at the 5% significant level and also look out for clustering in the VaR values. We selected the best fit models from Tables 11 and 12 both for the single- and regime-switching for this test. Table 18 displays the results of the DQ test for the models.

Table 18: DQ test result for both single- and double-regime models

	Model	p-value
GSE	GARCH-sged	0.0244
	MS2-EGARCH-sged	0.1519
KSE	GARCH-sstd	0.7883
	MS2-GARCH-ged	0.8026
NSE	EGARCH-sged	0.0370
	MS2-EGARCH-sged	0.0251
BSI	GARCH-sged	0.0728
	MS2-GARCH-sstd	0.3791

**MS2 – refers to the double-regime models*

With the exception of the NSE, the test statistics for the regime-switching models indicate that the null hypothesis for each of the bourses cannot be rejected at the 5% level. That is, the regime switching model performs significantly better than the single regime approaches in all the markets studied. NSE's backtest failure could be due to the extreme negative returns of -26.39% on 21-06-2016 (see Figure 19).

Remark 3

The code snippet to run the analysis Table in 18 is reproduced in Appendix B.

The performance of the models is shown in the VaR plot of Figures 17-20.

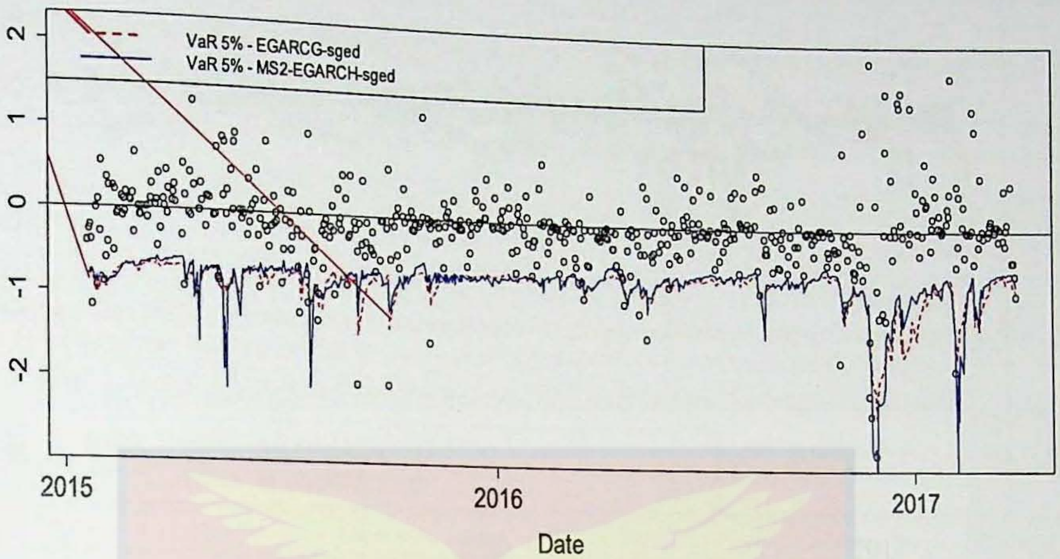

Figure 17: One-day VaR at 5% for sged E-GARCH (1,1) and 2-regime sged E-GARCH (1,1) for GSE

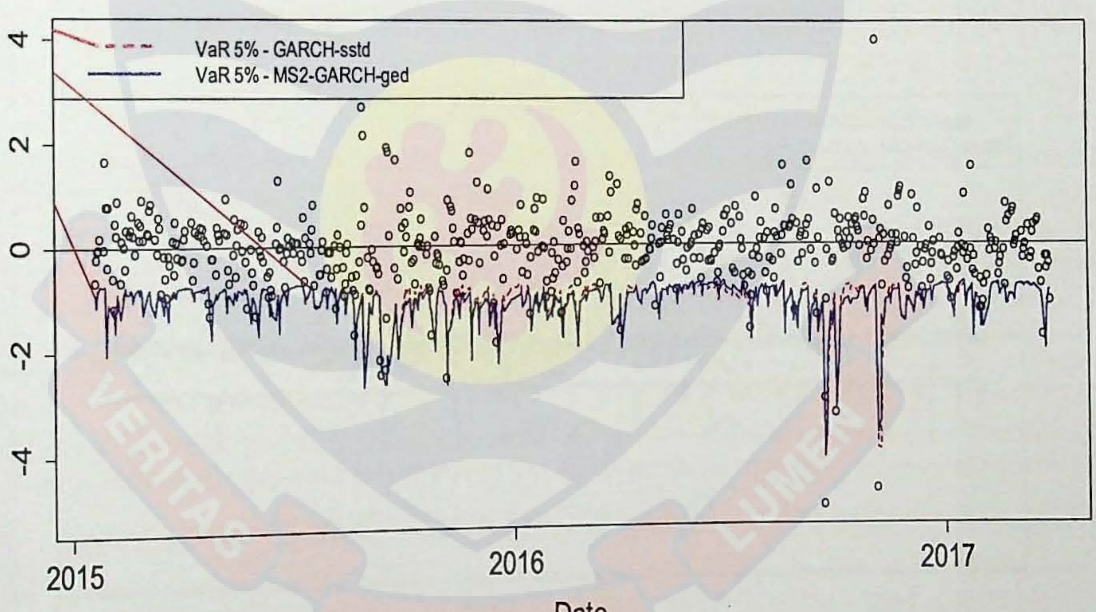

Figure 18: One-day VaR at 5% for sstd GARCH(1,1) and 2-regime sstd GARCH(1,1) for KSE

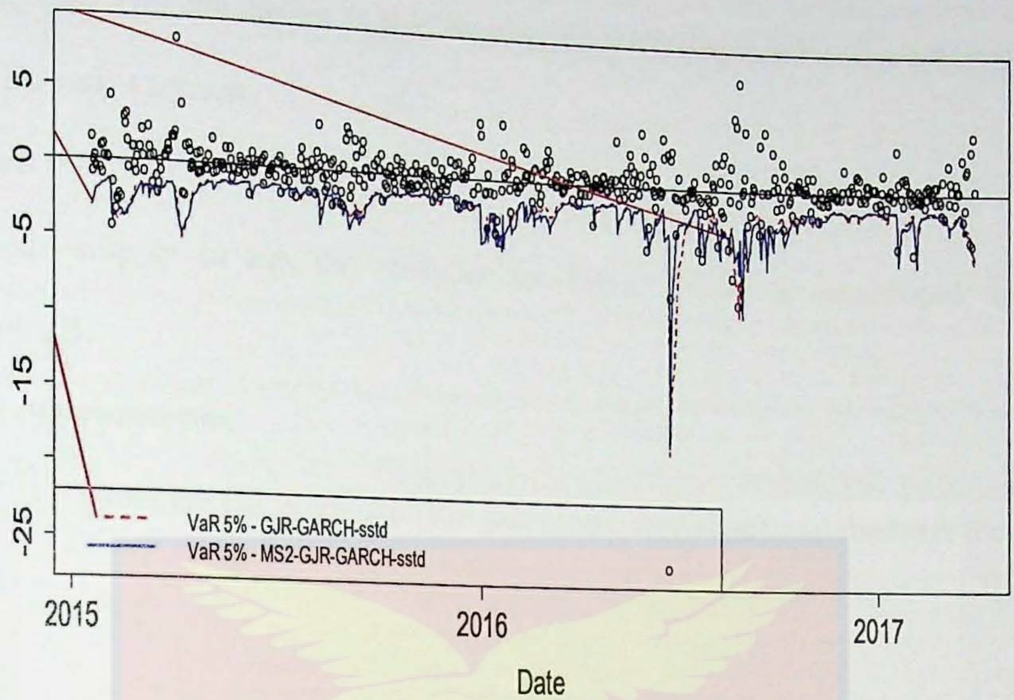

Figure 19: One-day VaR at 5% for Student-t GJR-GARCH (1, 1) and 2-regime Student-t GJR-GARCH (1, 1) for NSE

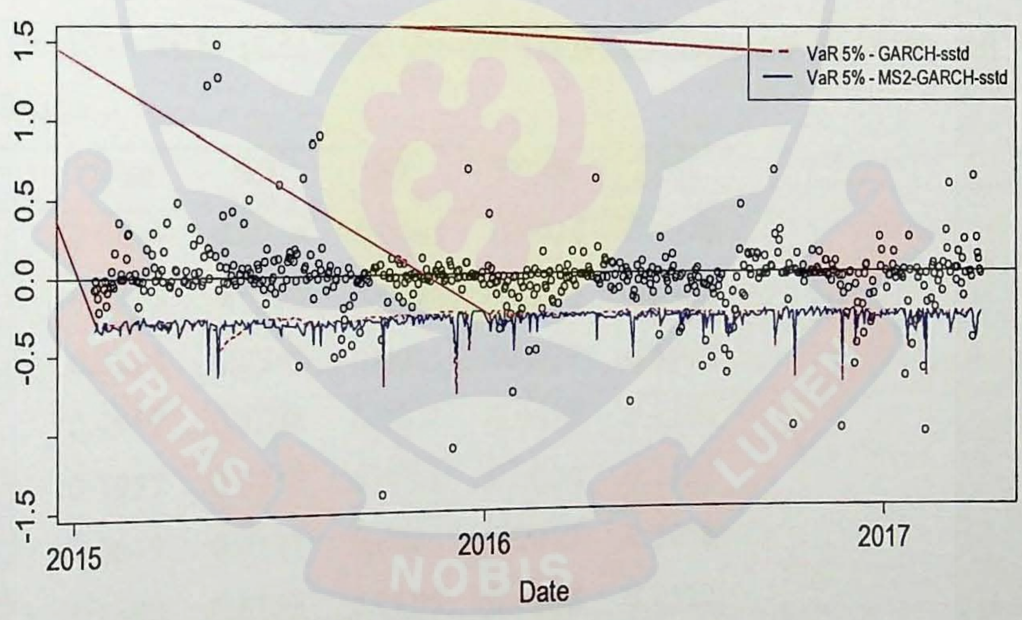

Figure 20: One-day VaR at 5% for Student-t GARCH (1, 1) and 2-regime Student-t GARCH (1, 1) for BSI

In the VaR plots of **Figures 17-20**, we see that the regime-switching (blue lines) provide a better description of the returns than the single-regime (red lines) models. Again, we see that the exceptions in the figures are independent. The conclusion is that the regime-switching models are a better fit for the index returns.

Remark 4

The code snippet to run the analysis for Figure 17-20 is reproduced in Appendix B.

Model Interpretation

We now take the models of the individual exchanges and interpret the estimates.

Ghana

Across the two regimes, the estimates indicate heteroscedastic behaviour in each regime. The estimates of the models' parameters are exhibited in Table 19.

Table 19: EGARCH (1,1) with skewed GED estimates for the GSE All Share Index

	Mean	SD	SE	TSSE	RNE
α_{01}	0.0809	0.0224	0.0007	0.0021	0.1172
α_{11}	0.153	0.0382	0.0012	0.0033	0.136
α_{21}	0.0283	0.0341	0.0011	0.0033	0.1072
β_1	0.4912	0.1036	0.0033	0.0099	0.1101
ν_1	0.7957	0.0357	0.0011	0.0024	0.2127
ξ_1	1.0649	0.0121	0.0004	0.0007	0.3054
α_{02}	1.1380	0.6163	0.0195	0.0538	0.1312
α_{12}	0.1519	0.0809	0.0026	0.0066	0.1517
α_{22}	0.0001	0.0000	0.0000	0.0000	0.0939
β_2	0.3753	0.1819	0.0058	0.0134	0.1835
ν_2	1.0501	0.4079	0.0129	0.0406	0.1012
ξ_2	0.7758	0.1302	0.0041	0.0108	0.1455
P_{11}	0.9915	0.0051	0.0002	0.0004	0.1959
P_{21}	0.0801	0.0423	0.0013	0.0024	0.3032

First, it is to be noted that the effects of leverage are largely pronounced in the GSE with its two-regime EGARCH (1, 1). Again, the tails are heavy. This implies there are tendencies to experience extreme returns in the market. The unconditional volatility of the low and high regimes is 7.40% and 15.71% respectively. The tail weights of 0.7957 and 1.0501 for regimes 1 and 2 respectively suggest the presence of fat-tails. This is supported by the skewed GED innovations in the model. We also see different negative reaction, $\alpha_{12} = 0.0283$ for the low volatility regime and $\alpha_{22} = 0.0001$ for the high volatility regime. The persistence of the volatility is different for both regimes. It is $\alpha_{11} + \frac{1}{2}\alpha_{12} + \beta_1 = 0.6554$ for the low volatility regimes and $\alpha_{12} + \frac{1}{2}\alpha_{22} + \beta_2 = 0.5273$ for the high volatility regime. Volatility therefore dissipated slowly in regime 2.

Thus, we conclude for regime 1 that it is of low unconditional volatility, high negative reaction to past volatility and high persistence in volatility. For the high volatility regime, we summarised it as having a high unconditional volatility, a mild reaction to past negative returns and a volatility process with a low persistence.

The transition matrix is shown in Table 20. The states stay in each regime longer before transitioning into another regime. The period of transition is rather short. That could be due to the abrupt changes in the underlying dynamics driving the GSE.

Table 20: Posterior mean transition matrix for the GSE

	t+1|k=1	t+1|k=2
t|k=1	0.9915	0.0085
t|k=2	0.0801	0.9199

The transition matrix shows that the volatility spends almost equal time in low and high volatility regime. The transition across regimes is rather slow.

The evolution of the volatility over the sample period for the GSE returns is shown in Figure 21.

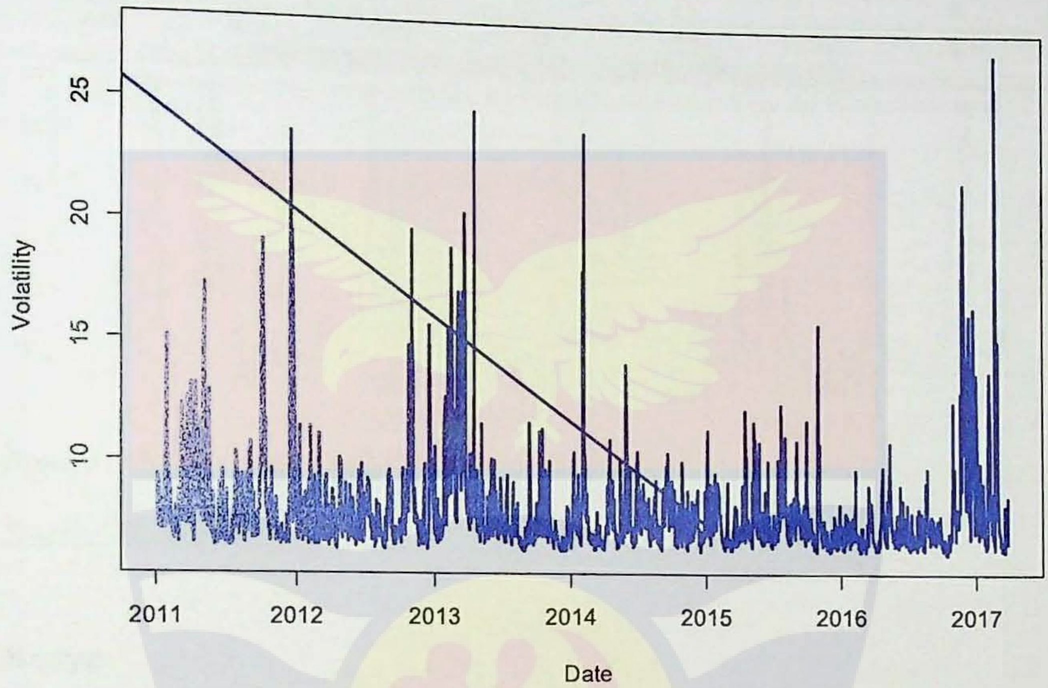

Figure 21: Evolution of volatility over the sample period for GSE returns019)

Figure 21 reveals that much of the period 2011-2012 was largely volatile. Apart from that, in the period that followed 2012, the market has been volatility at the end of the year into the beginning of the following year. We see again a relatively serene volatility period in the second half of 2014 to the third quarter of 2016.

The smooth probability plot of the weekly resampled median for the GSE returns in **Figure 22** show the pattern of the low volatility regime being interrupted with high volatility regimes of rather short but frequent periods. This shows investors could be caught unawares for brief periods of trading.

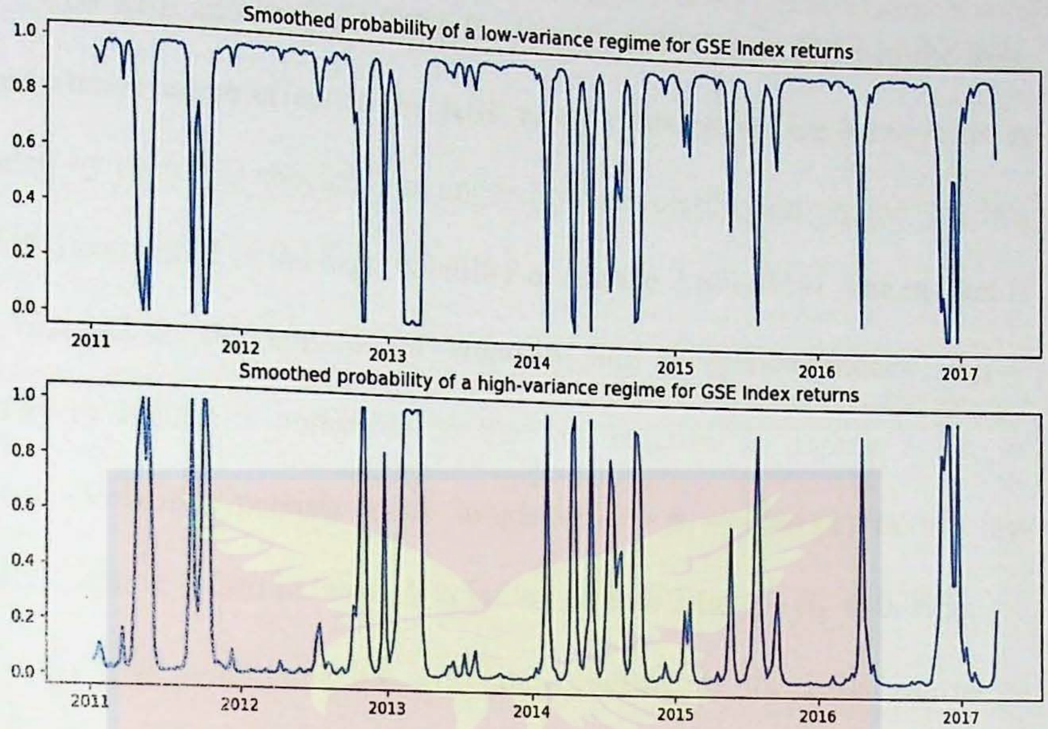

Figure 22: Smoothed probability plot for GSE

Source: Korkpoe (2019)

Kenya

The parameter estimates for the KSE is shown in Table 21.

Table 21: GARCH (1,1) with GED estimates for the KSE All Share Index

	Mean	SD	SE	TSSE	RNE
α_{01}	0.138	0.0306	0.001	0.0013	0.5869
α_{11}	0.4219	0.0637	0.002	0.003	0.4635
β_1	0.2192	0.0634	0.002	0.0035	0.3344
ν_1	1.5491	0.1213	0.0038	0.0056	0.4696
α_{02}	0.8952	0.2502	0.0079	0.0164	0.2333
α_{12}	0.0541	0.0411	0.0013	0.0017	0.5536
β_2	0.0409	0.082	0.0026	0.0057	0.2054
ν_2	1.0363	0.1901	0.006	0.01	0.364
P_{11}	0.8627	0.0791	0.0025	0.0043	0.3369
P_{21}	0.904	0.1436	0.0045	0.0091	0.2479

The KSE admits a GARCH (1, 1) with GED innovations in the tails. There is no leverage effect in the KSE returns though we see heavy-tails as modeled by the GED shocks. The unconditional volatility on regime 1 is low (10.41%) compared to the high volatility of regime 2 (49.81%). The market is very reactive to the conditional volatility due to market shocks ($\alpha_{11} = 0.4219$) in regime 1 compared to the low reaction in regime 2 ($\alpha_{12} = 0.0541$). Volatility persists much longer ($\alpha_{11} + \beta_1 = 0.6411$) in the low volatility regime 1 than in the high volatility regime 2 ($\alpha_{12} + \beta_2 = 0.095$).

We can summarise the evolving volatility of the KSE returns as showing two heterogeneous regimes with a relatively low and a very high heteroscedastic cycles; a low volatility regime very reactive to negative returns and a high volatility regime with a very low persistence in conditional volatility. The persistence which is less than 0.9 in each regime indicates that volatility subsides very quickly.

The multistate transition from one state to the other is shown in Table 22. The state of the market in the stable regime 1 takes before transitioning into regime 2. The market actually spends little time in regime 2. This is a market that is mostly serene.

Table 22: Posterior mean transition matrix for the KSE

	t+1|k=1	**t+1|k=2**
t|k=1	0.8627	0.1373
t|k=2	0.904	0.096

The volatility in KSE is for the most part in regime 1 with a probability of 0.8627. The time spent in regime 2 is almost transitory with a probability of 0.096. Transition from a high to low regime has a probability of 0.904 whereas from a low regime to high regime is slow with a probability of 0.1373.

The evolution of the volatility over the sample period for the KSE returns is shown in **Figure 23**. The KSE has been particularly volatile towards the end of 2016. The period 2011 into 2012 similarly have been volatile except for a brief lull in mid-2011. The year 2012 was mostly quiet for the returns whereas the market was turbulent in 2013. There does not seem to be a pattern to the evolution of volatility on the KSE. This is what investors must pay attention to the risks in the market as abrupt switches in volatility can cause distress in the market.

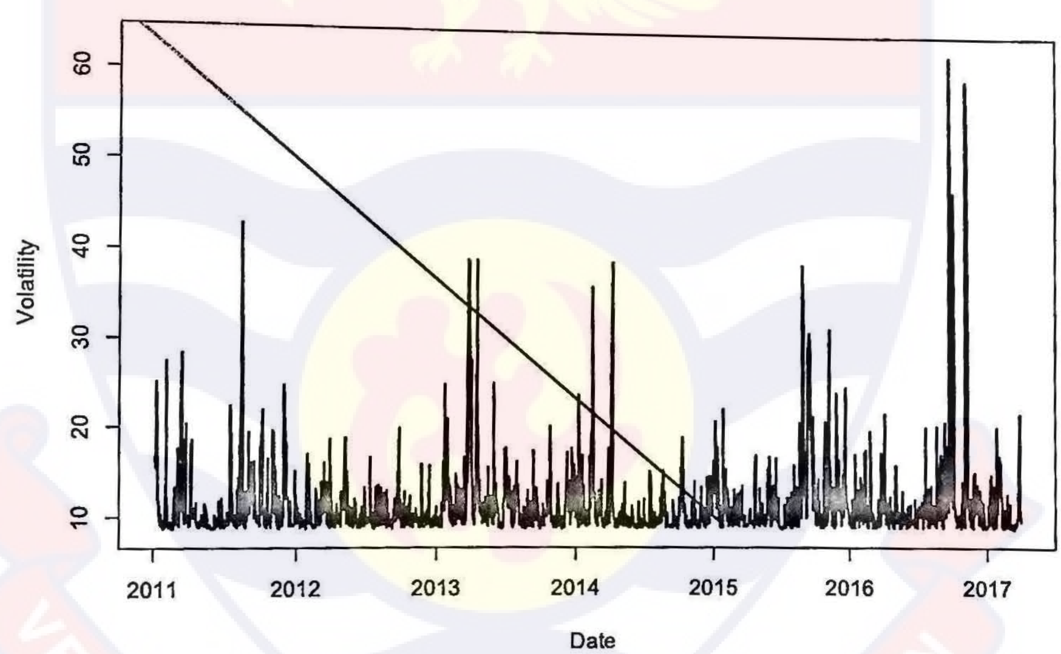

Figure 23: Evolution of volatility over the sample period for KSE returns

Source: Korkpoe (2019)

The KSE smoothed probability plot in **Figure 24** shows a mirror image of the preponderance of low versus high volatilities. There is quick switching in the regimes and this could be due to the domination of financial and banking services firms with risky stocks very sensitive to policy changes. Elyasiani and Mansur (1998), Akella and Chen (1990) and more recently

Kurov (2010) identified bank and financial services related stock as having a heightened response to a dynamic monetary policy environment. Kenya has been known to keep calibrating their fiscal and monetary policies in response to global and local financial developments (Andrle, Berg, Morales, Portillo, & Vlcek, 2015)

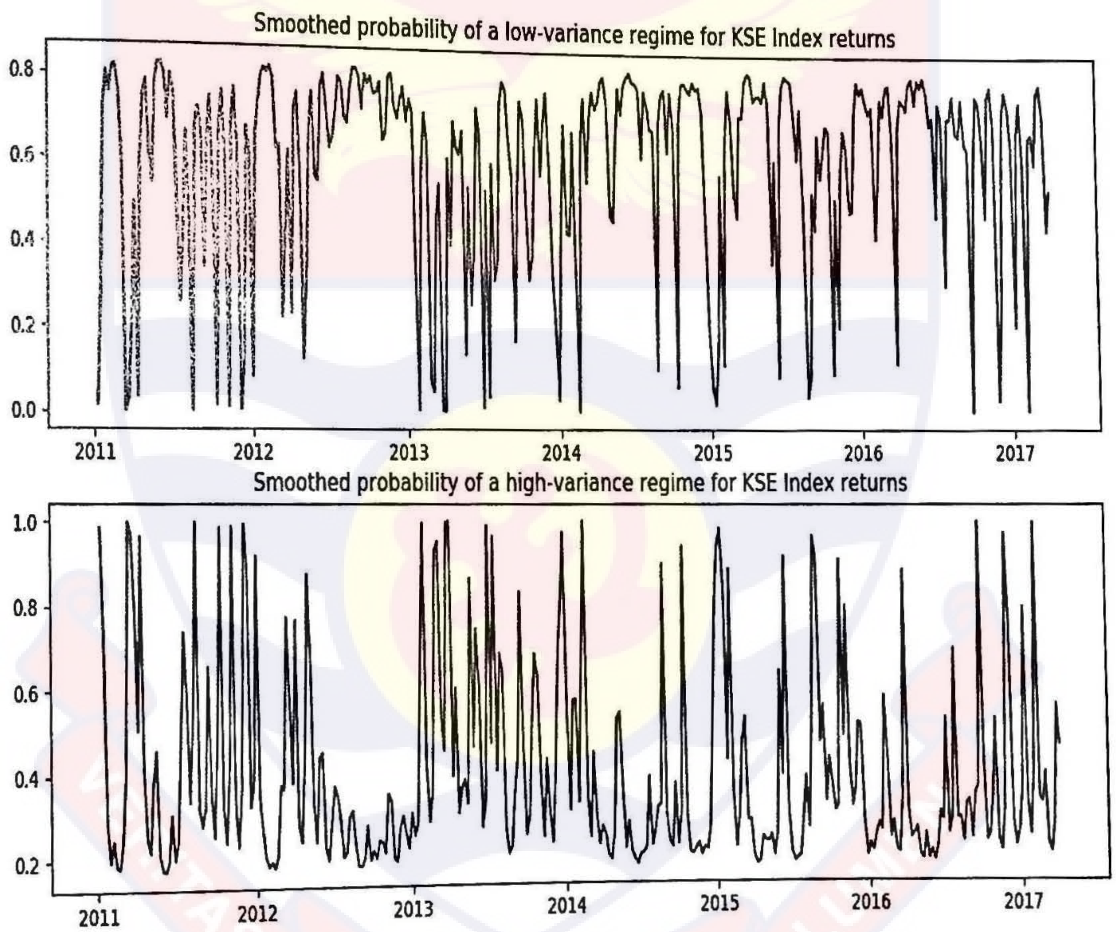

Figure 24: Smoothed probability plot for KSE

Nigeria

The GJR-GARCH (1, 1) with skewed Student's *t* innovations fits the returns of the NSE. This indicated a fat-tailed distribution with negative shocks having more pronounced effect in conditional volatility than positive innovations. The model also admits leverage effects. The parameter estimates are shown in Table 23.

Table 23: GJR-GARCH (1, 1) estimates with skewed Student's t innovations for the NSE All Share Index

	Mean	SD	SE	TSSE	RNE
α_{01}	0.3264	0.6508	0.0206	0.0328	0.3943
α_{11}	0.3456	0.0854	0.0027	0.0051	0.2838
α_{21}	0.021	0.0228	0.0007	0.0013	0.3192
β_1	0.5191	0.0926	0.0029	0.007	0.1729
ν_1	4.631	8.6814	0.2745	0.4835	0.3224
ξ_1	0.9659	0.0781	0.0025	0.0032	0.5836
α_{02}	5.1929	5.9048	0.1867	0.8106	0.0531
α_{12}	0.8099	0.1676	0.0053	0.0111	0.2289
α_{22}	0.0018	0.0054	0.0002	0.0005	0.1409
β_2	0.0113	0.0661	0.0021	0.0036	0.341
ν_2	55.4033	25.6674	0.8117	2.3201	0.1224
ξ_2	0.9862	0.3944	0.0125	0.0582	0.0459
P_{11}	0.9933	0.0179	0.0006	0.0007	0.6664
P_{21}	0.1194	0.1576	0.005	0.0209	0.057

The unconditional volatility regimes 1 and 2 respectively is 16.58% and 44.69% indicating regime 1 is the low volatility regime and regime 2 the high volatility regime. The values ξ_1 and ξ_2 are both greater than 0 indicating leverage effects in the returns. The tails in regime 1 are leptokurtic ($\nu_1 = 4.631$) compared to regime 2's tails ($\nu_2 = 55.4033$). The market's response to negative returns in regime 1 is swifter ($\alpha_{21} = 0.021$) than to conditional volatility in regime 2 ($\alpha_{22} = 0.0018$). The persistence of volatility of both regimes appears roughly the same - $\alpha_{11} + \frac{1}{2}\alpha_{21} + \beta_1 = 0.8752$ and $\alpha_{12} + \frac{1}{2}\alpha_{22} + \beta_2 = 0.8221$ for regime 1 and regime 2 respectively.

Table 24 shows the transition matrix for the NSE. The market is in either regime for long periods with short transitioning times in between periods.

Table 24: Evolution of volatility over the sample period for NSE returns

	t+1|k=1	t+1|k=2
t|k=1	0.9933	0.0067
t|k=2	0.1194	0.8806

Volatility alternates almost equally in NSE between low (regime 1) and high regimes (regime 2) and spends little time transitioning from the low regime to the low regime. The reverse is true for the transition from the high regime 2 to the low regime 1.

The high volatility regimes are moderate with the exception of mid-2016 when the market took a serious dive briefly. This is show in **Figure 25**.

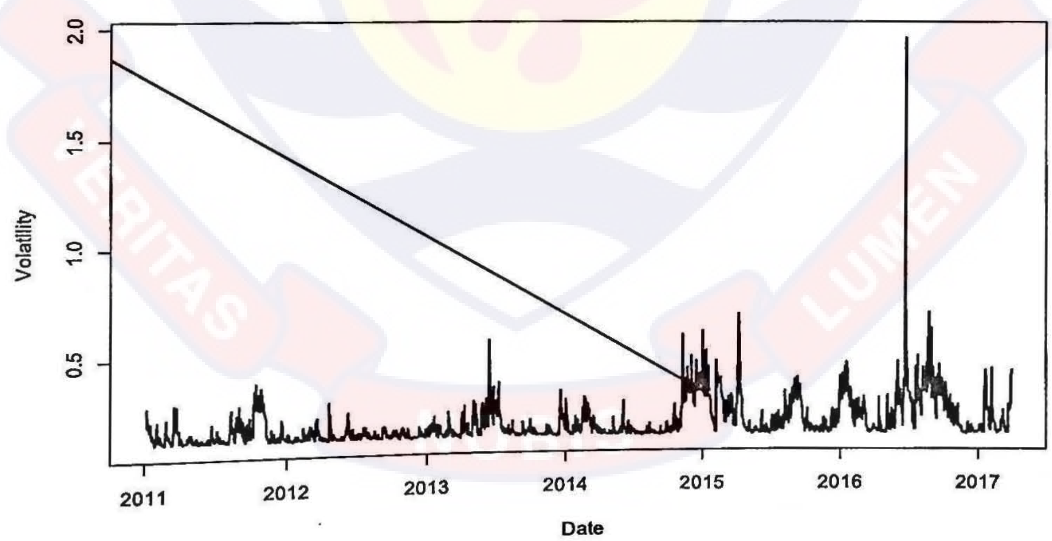

Figure 25: Evolution of volatility over the sample period for NSE returns

Basically, the NSE bourse for most periods is in the low regime as shown in **Figure 26**. From 2011 to the latter part of 2014, there have been two albeit brief spikes in volatility in the market. However, the market has been more turbulent swinging between low and high volatility with no clear dominance of each of these regimes. For most part, the NSE, apart from the market crash of 2008, is well diversified and less responsive to global developments. Uduanu (2019) points out that retail investors have been cautious since the NSE market crash. One can infer therefore that the bullish retailers that swing the market up and down very often have given way to other moderating factors of volatility swings in the market.

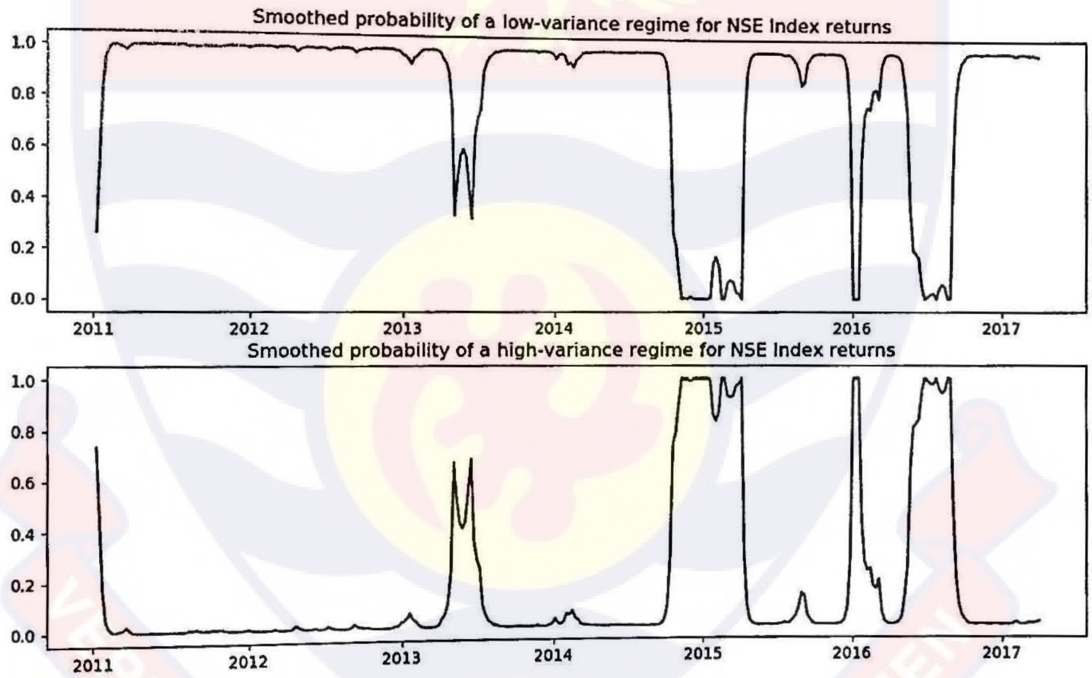

Figure 26: Smoothed probability for NSE

Botswana

GARCH (1,1) with skewed Student's *t* innovations provide the best fit for the returns of the BSI index. The parameter estimates are shown in Table 25.

Table 25: GARCH (1, 1) with skewed Student's t estimates for the BSI All Share Index

	Mean	SD	SE	TSSE	RNE
α_{01}	0.0005	0.0002	0.0000	0.0000	0.0939
α_{11}	0.0013	0.0020	0.0001	0.0002	0.1147
β_1	0.3954	0.1570	0.0050	0.0166	0.0900
ν_1	96.5569	3.9439	0.1247	0.4185	0.0888
ξ_1	2.9389	2.5134	0.0795	0.6672	0.0142
α_{02}	0.3295	0.0409	0.0013	0.0030	0.1799
α_{12}	0.7090	0.1204	0.0038	0.0111	0.1185
β_2	0.0002	0.0001	0.0000	0.0000	0.1519
ν_2	2.1069	0.0082	0.0003	0.0009	0.0884
ξ_2	1.0488	0.0238	0.0008	0.0016	0.2273
P_{11}	0.4597	0.0751	0.0024	0.0061	0.1526
P_{21}	0.1969	0.0285	0.0009	0.0019	0.2308

The BSI is a relatively quiet stock exchange with unconditional volatilities of 2.08% and 6.86% for regime 1 and regime 2 respectively. The GARCH error for regime 1, $\alpha_{11} = 0.0013$, indicates a lack of sensitivity of volatility to market events in contrast to that of regime 2 which is $\alpha_{12} = 0.7090$. The conditional lag parameter for regime 2, $\beta_2 = 0.0002$, is swift compared to that of the low regime of $\beta_1 = 0.3954$. There is a leptokurtic tail ($\nu_2 = 2.1069$) for the second regime relative to that of regime 1 of $\nu_1 = 96.5569$. The rate of convergence of the conditional volatility to average long-term levels is relatively low, i.e. $\alpha_{11} + \beta_1 = 0.3967$ and $\alpha_{12} + \beta_2 = 0.7092$ for the low and high regime respectively.

The transition matrix in Table 26 shows how volatile the BSI market has evolved over the sample period. The market spends almost the same period in each of the state and has a higher probability of transitioning from one state to the other.

Table 26: Evolution of volatility over the sample period for BSI returns

	t+1|k=1	t+1|k=2
t|k=1	0.4597	0.5403
t|k=2	0.1969	0.8031

In the BSI, the volatility spends more time in regime 2 with a probability of 0.8031 compared to the low probability regime of probability 0.4597. The transition from regime 1 to regime 2 is swifter with a probability of 0.5403 compared with the reverse with a probability of 0.1969 in transitioning from regime 2 to regime 1.

The volatility instability of the BSI is clearly shown in **Figure 27**. The volatility is wavy with transitions from one state to the other occurring with rapid frequency. It is not immediate immediately clear what is driving this dynamic.

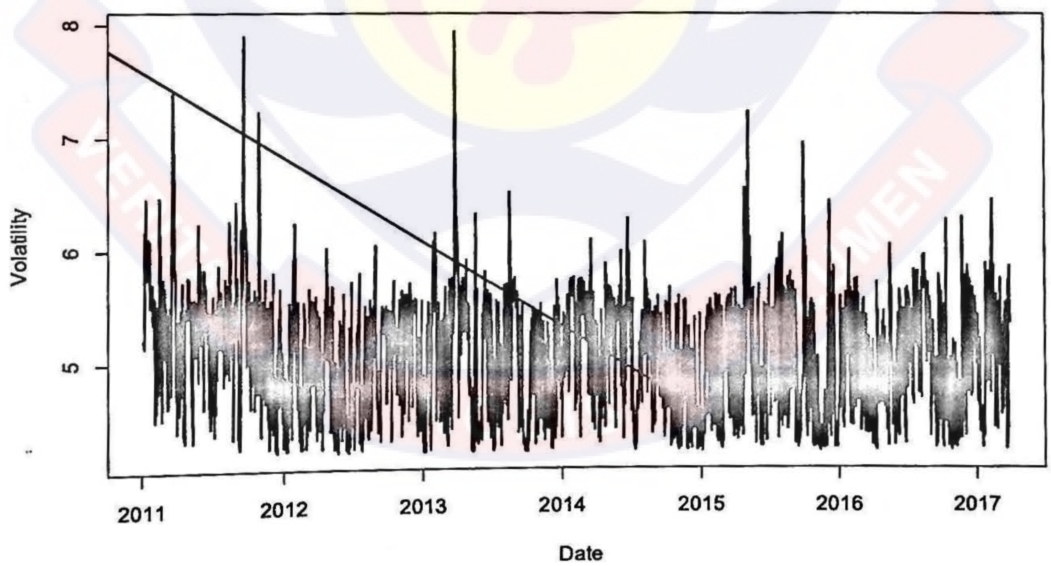

Figure 27: Evolution of volatility over the sample period for BSI returns

The BSI is a relatively small market but lists a lot of investors on their market from other parts of the world and therefore more likely to be buffeted by developments on the global financial markets. As shown in **Figure 28**, the switch from low to high volatility regimes occurs with almost rapidly. Indeed, most foreign firms on the BSI have dual listing on the local and outside markets like London, Johannesburg, Toronto and Australia. The regularity of the market swings on the BSI could be a reflection of markets outside of Botswana.

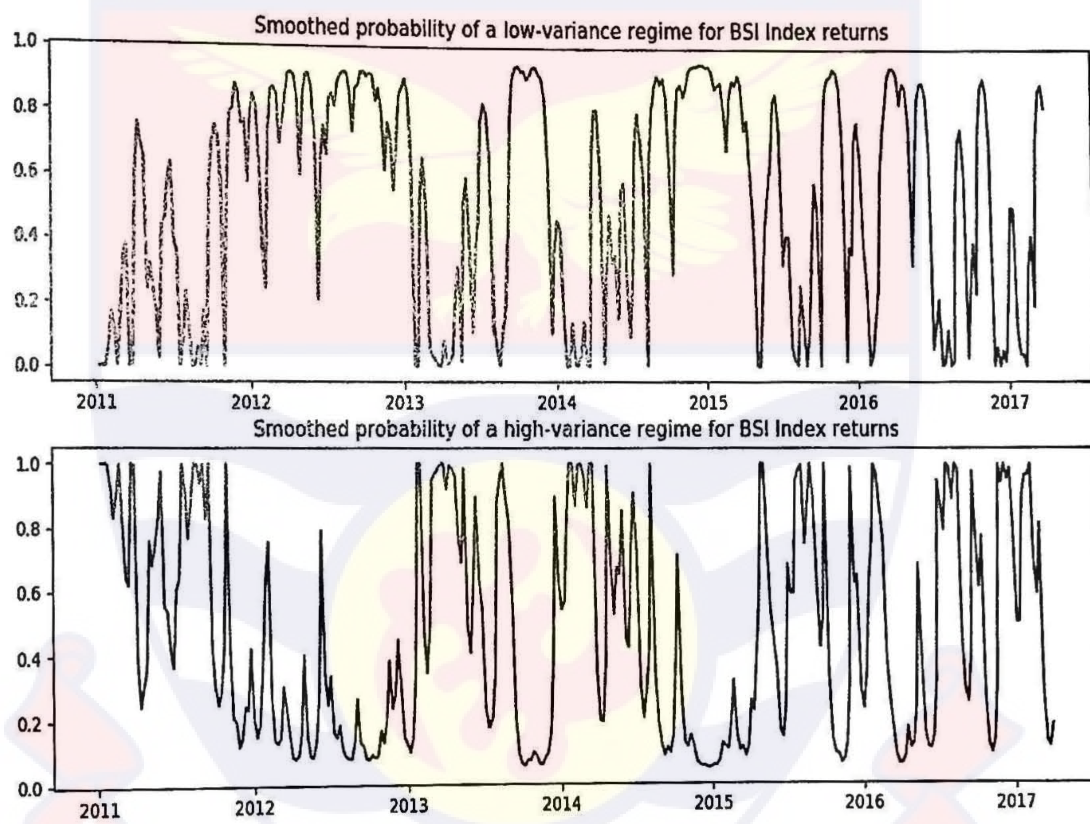

Figure 28: Smoothed probability plot of BSI

Dependencies and Tail Behaviour among Exchanges

For all the exchanges and their modeled heteroscedastic functions, we have seen a heterogeneous behaviour of the volatility across markets and regimes. That is an indication of the differences in the underlying data generating mechanisms at work in each of these economies. Evidence of this is the low correlations of returns of these exchanges as shown in Table 27.

Table 27: Correlations among the returns of the exchanges

	GSE	KSE	NSE	BSI
GSE	1			
KSE	0.0949	1		
NSE	0.0002	0.0281	1	
BSI	0.0313	0.0212	0.0265	1

Increasingly, assets markets are becoming interconnected by the day as a result of globalisation powered by interconnectivity. However, trade among the frontier markets still remains very low. Their exchanges are insulated from each other. Table 27 is a proof that equity markets of sub-Saharan Africa are a disparate collection given the low correlation among them. There are probably different forces driving the volatility in these countries. This lack of any co-movements across the markets could also be an indication of lack of any significant economic or political related development in the sub-Saharan region. The particularly low correlation between GSE and NSE, two markets in the West African sub-region, needs to be investigated further.

Furthermore, all the returns exhibit heavy-tails. Some authors, for example, Jones, Walker and Wilson (2004) and Turner and Weigel (1992) used 1.5 standard deviations from the lower and upper quantiles to characterise extreme deviations in their work. Here we use returns beyond ± 3 standard deviations from the mean of the returns as a measure of extreme outcomes.

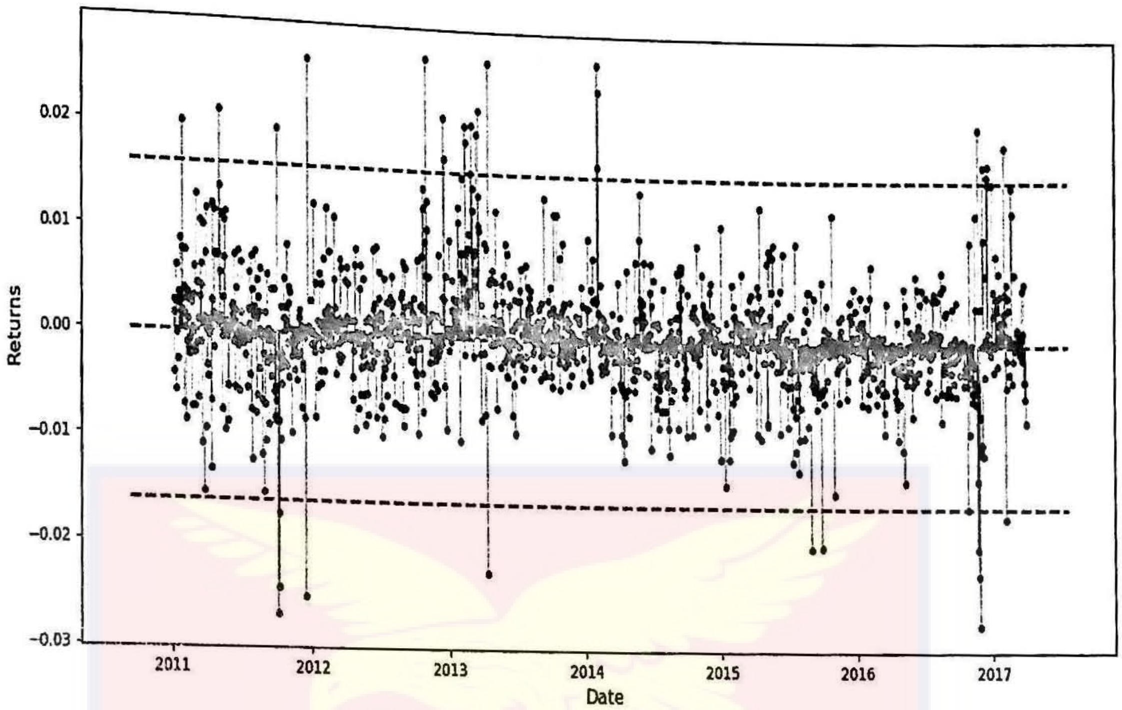

Figure 29: Extreme outcomes beyond three standard deviations for GSE Returns

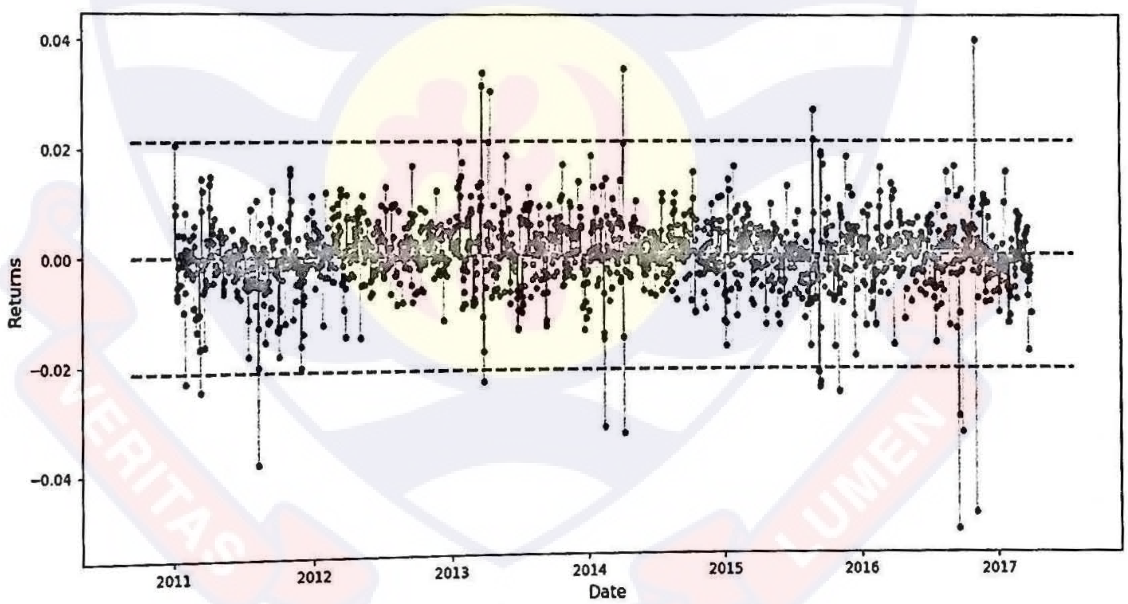

Figure 30: Extreme outcomes beyond three standard deviations for KSE Returns

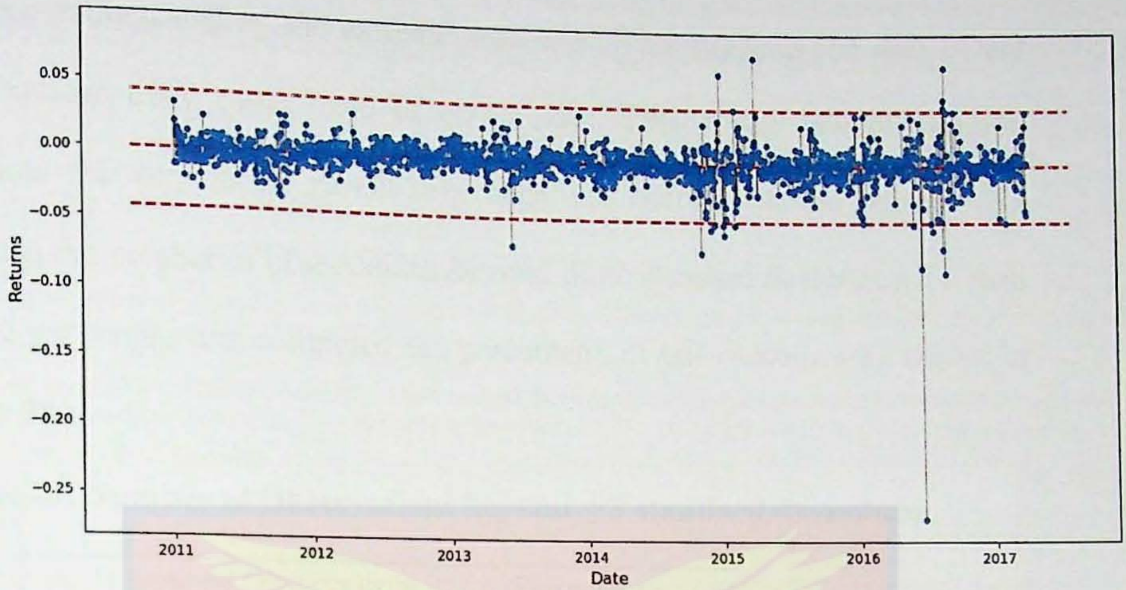

Figure 31: Extreme outcomes beyond three standard deviations for NSE Returns

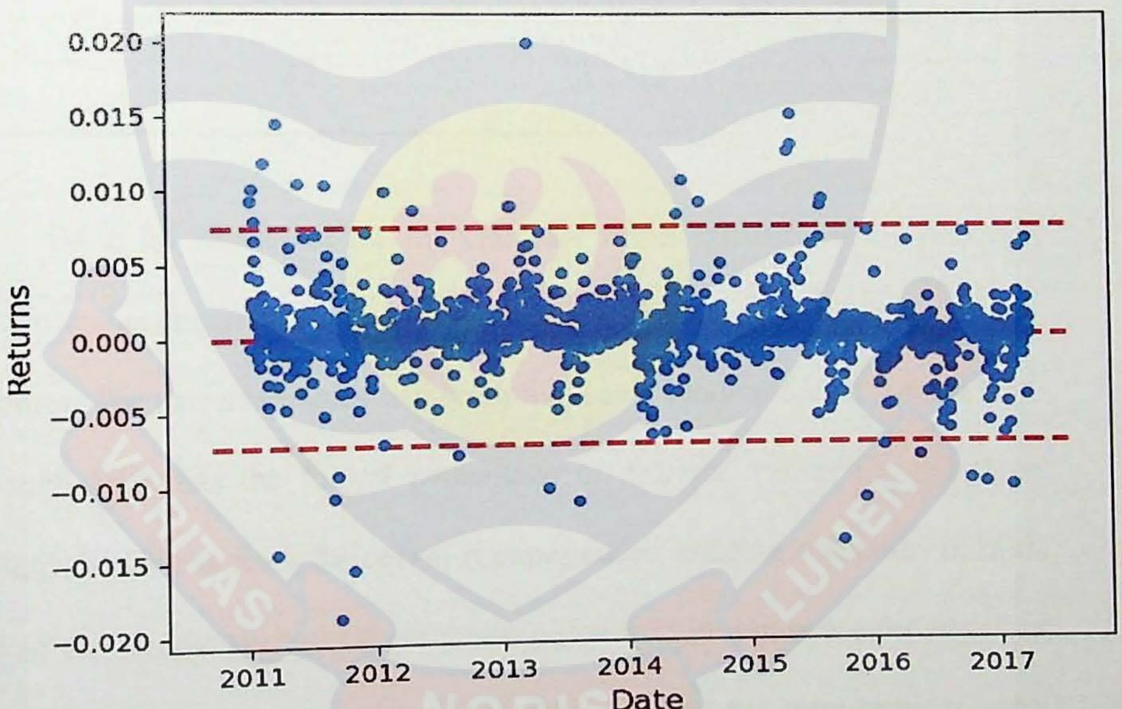

Figure 32: Extreme outcomes beyond three standard deviations for BSI Returns

Figures 29-32 show the returns with the horizontal broken lines depicting ± 3 standard deviations from the mean. As the figures show, the distributions though are tighter around the mean, are heavy-tailed to varying degrees as indicated by the extreme outcomes. This suggests the tails of the distributions obey some form of power law. This power law distribution explains the very large, particularly negative, changes in the returns. We counted the number of observations beyond three standard deviations for each of the exchanges and computed the percentage of tail outcomes as shown in Table 28.

Table 28: Number of Observations beyond ±3 standard deviations

Exchange	Count of Observations	Percentage
GSE	29	1.8733
KSE	19	1.2274
NSE	14	0.9044
BSI	33	2.1318

Table 28 shows all the markets saw some extreme values although there is no benchmark in percentage terms to determine the severity of such outcomes. The extreme values will seem like being random for all the markets. Although NSE has the lowest percentage of extreme values beyond three standard deviations from the mean, it experienced extreme outcomes in mid-2016 of the sample period. Of particular concern to investors are the negative returns and the diagram shows the GSE, KSE and BSI are more prone to such returns and therefore subject to tail risks. There is an average of 24 extreme negative outcomes across the markets of the entire region over the sample period with BSI recording the most extreme of such returns in the sample period.

Estimation of the VaR of the Exchanges

The summary statistics for the returns in Table 7 show the kurtosis of the distributions are all above three and giving rise to fat-tails. This is further confirmed by the graphs of Figure 29-32 depicting return values beyond ±3 standard deviations of the means. We show further how the extremities of the tails deviate from the Gaussian in the QQ plots of Figures 33–36. As the figures show, the deviation from normality is especially severe for GSE, KSE and BSI. There was one outlying lower tail return for NSE that skewed the returns for the exchange. The estimation of the VaR based on the underlying assumption of normality in the distributions is likely to lead to the underestimation of risk (Olson & Wu, 2013).

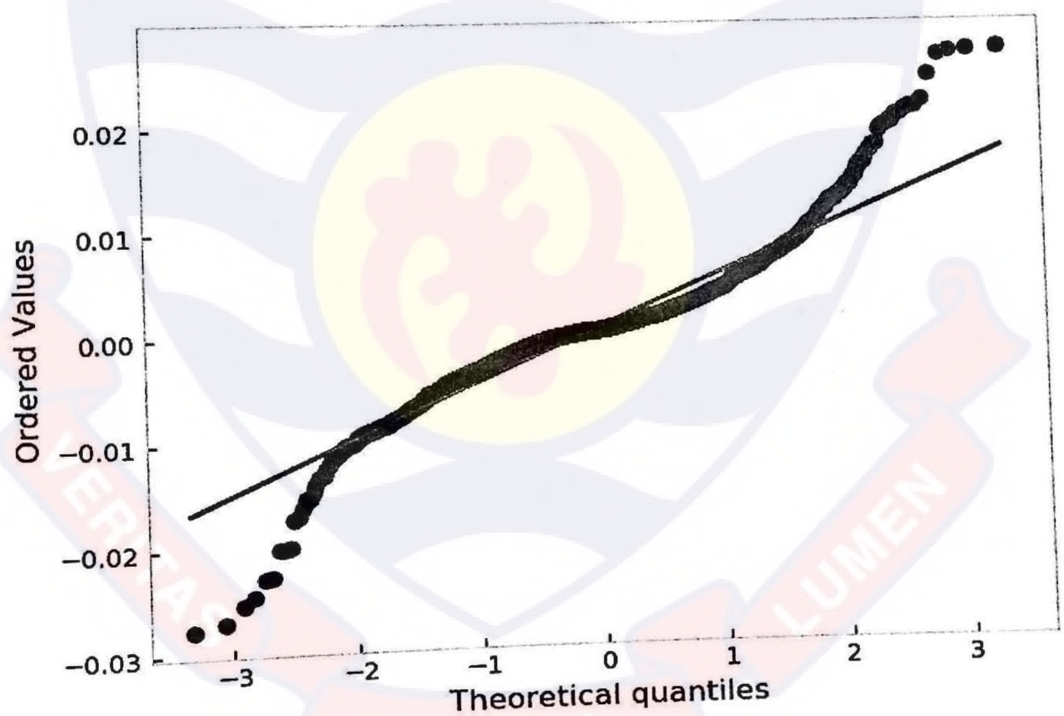

Figure 33: QQ plot for GSE Index returns

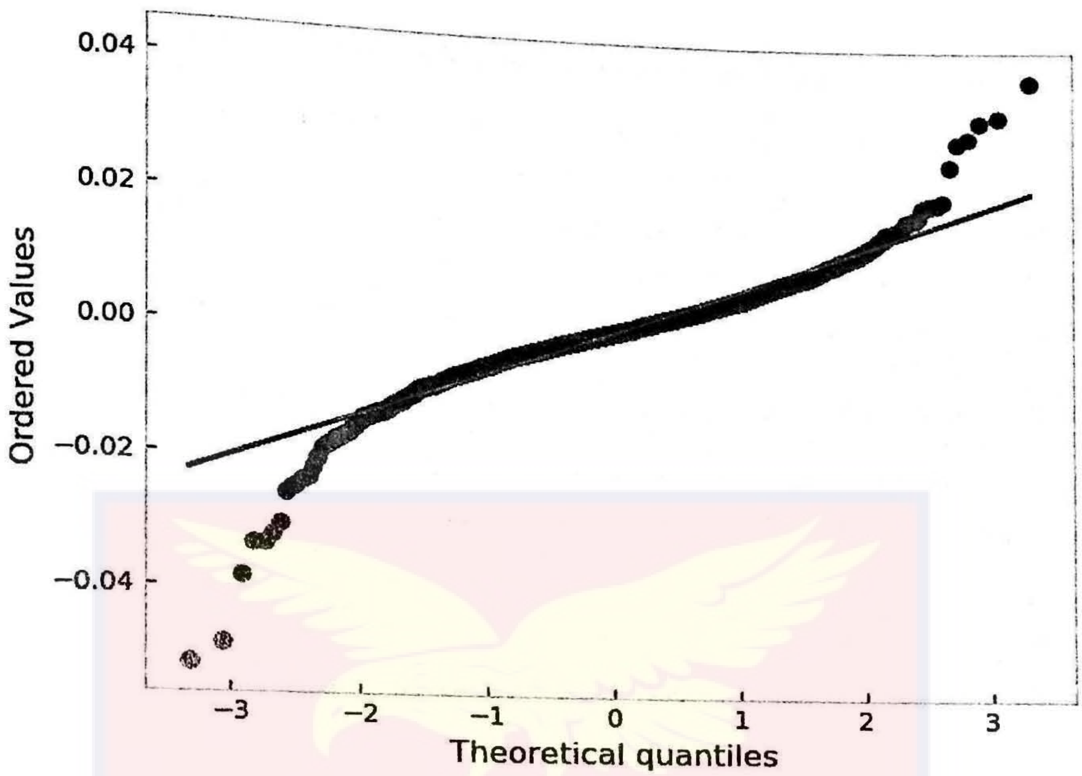

Figure 34: QQ plot of KSE Index returns

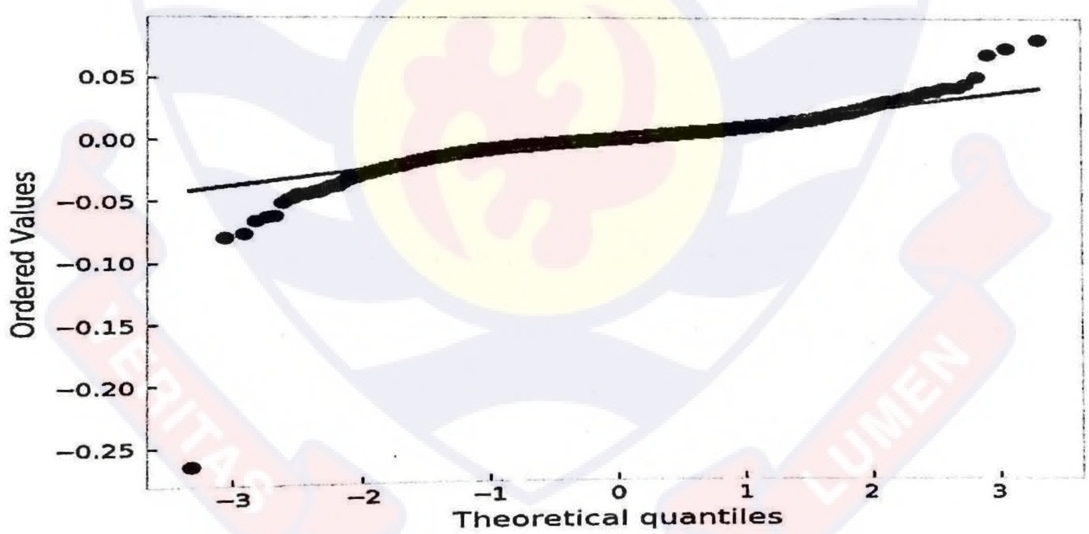

Figure 35: QQ plot of NSE Index returns

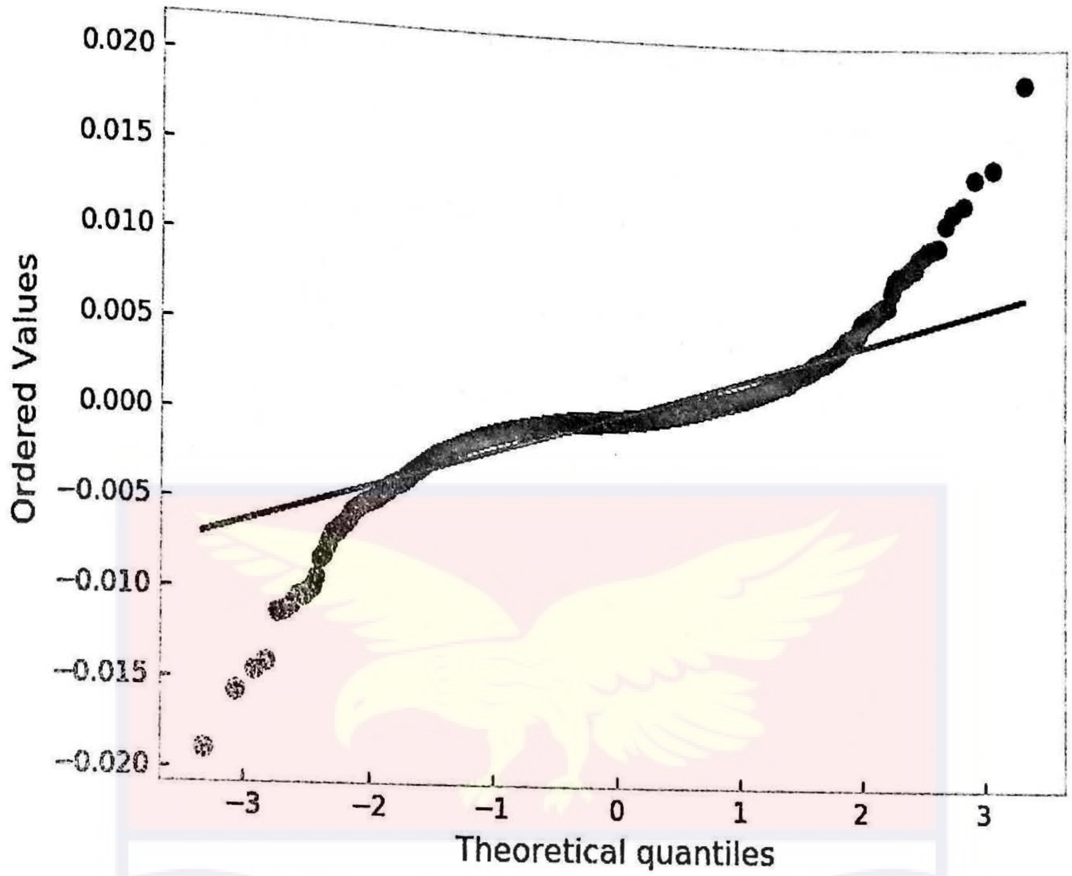

Figure 36: QQ plot for BSI Index returns

The 1-day VaR at the 90%, 95% and 99% based on the assumption of normality and the historical method was estimated in the absence of a sufficiently long data series to estimate with the computationally intensive methods which will capture the tail behaviour of the distributions (Günay, 2017; Glasserman, Heidelberger, & Shahabuddin, 2002). The findings on the behaviour of the VaR with respect to financial returns are in line with Galeano and Ausín (2010) and Harmantzis, Miao and Chien's (2006) that this particular risk measure under the assumption of normality underestimates the risk and can lure firms into complacency. This is particularly true for new markets such as frontier and some parts of the emerging markets (Gençay & Selçuk, 2004). Evidence in some frontier markets in the Middle East and North African region has been provided in Assaf (2009) which look at the estimation issues related to tail-risk measures. At the 99% level recommended by the Bank for International Settlements (2011), Table 29 shows the most conservative values for all the exchanges are those of the historical approach. The normal approach will appear too optimistic given the how fat the tails of the distributions are. The estimated Hill indices characteristing the degree of heavy-tailedness of the various exchanges are shown in Table 30. The values are all positive confirming the deviations from Gaussian distributions.

Table 29: VaR of the exchanges

	GSE		KSE		NSE		BSI	
	Normal	Historical	Normal	Historical	Normal	Historical	Normal	Historical
90%	-0.00647	-0.00508	-0.0089583	-0.0073	-0.01859	-0.01233	-0.00291	-0.00165
95%	-0.00842	-0.00792	-0.0115314	-0.01081	-0.02374	-0.01958	-0.0038	-0.00316
99%	-0.01208	-0.01373	-0.0163581	-0.01961	-0.03339	-0.04129	-0.00547	-0.00699

Table 30: Hill's estimator at 99%

GSE	KSE	NSE	BSI
0.051121	0.415426	1.989833	*

* Could not be estimated because did not meet threshold

Remark 5

The code snippet to run the analysis in Tables 29 and 30 is reproduced in Appendix B.

Month-on-month Volatility

We resampled the daily index returns to monthly outcomes and took the 12-month annualized volatility of the returns. The plot is shown in Figures 37-40 with the patches depicting periods of high volatility across the sample period.

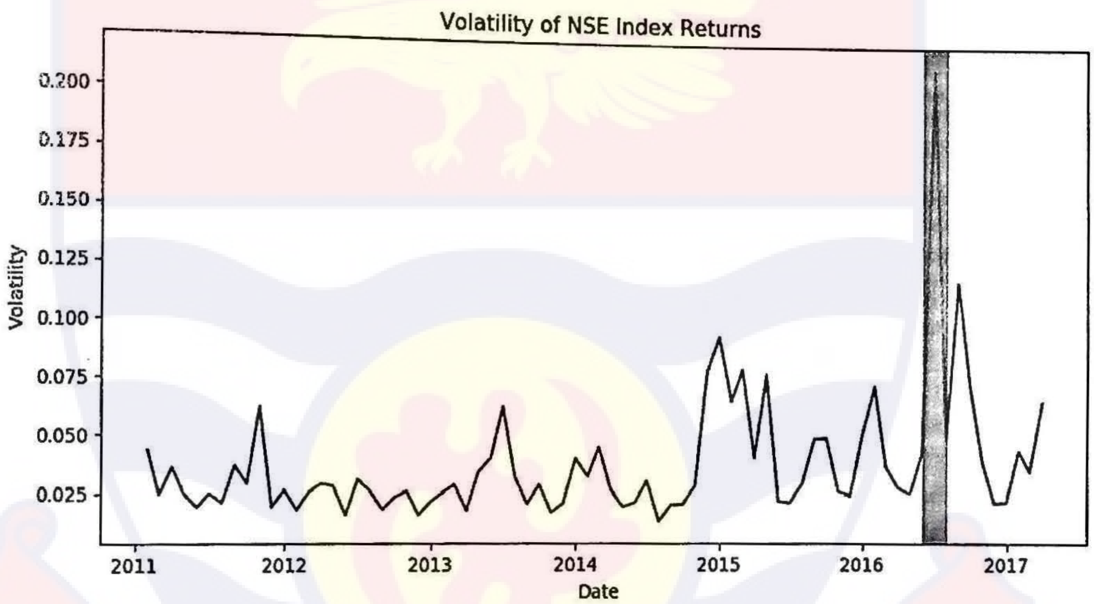

Figure 37: 12-month resampled volatility of NSE with patched regions showing high volatility

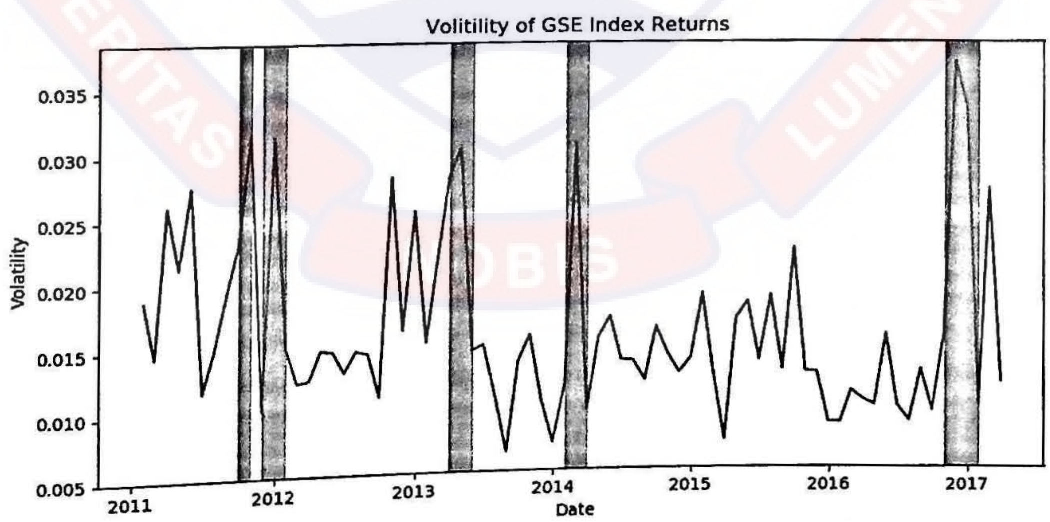

Figure 38: 12-month resampled volatility of GSE with patched regions showing high volatility

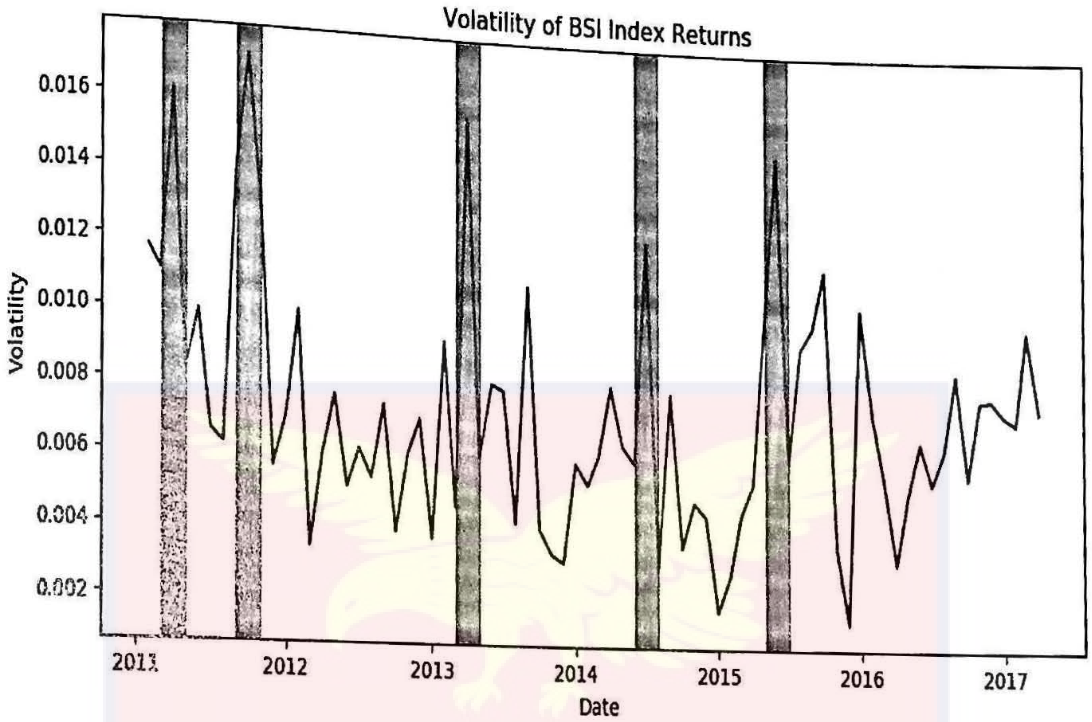

Figure 39: 12-month resampled volatility of BSI with patched regions showing high volatility

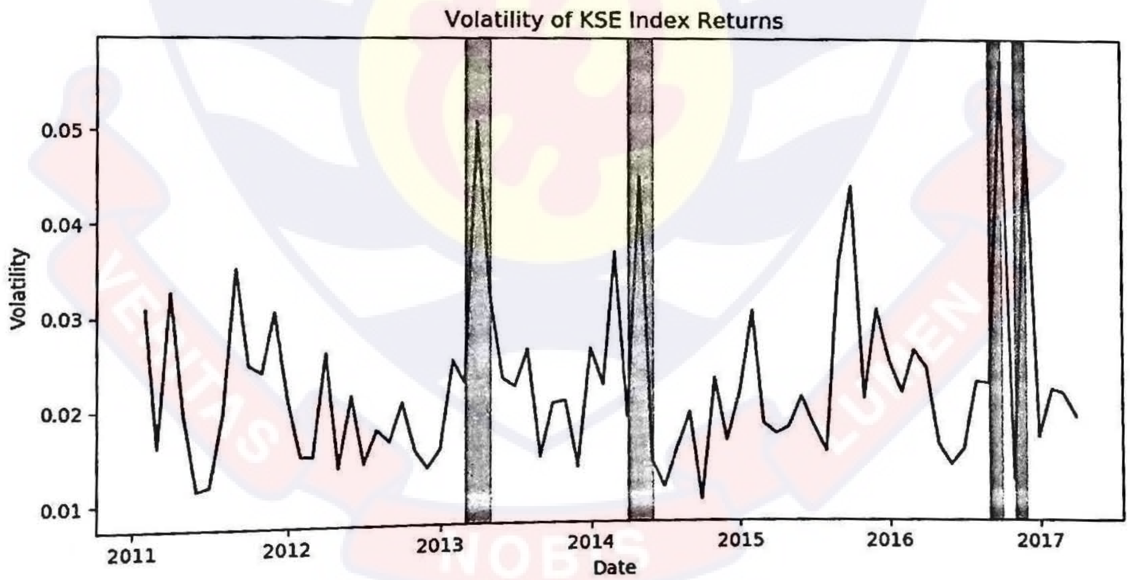

Figure 40: 12-month resampled volatility of KSE with patched regions showing high volatility

GSE appears more volatile of the exchanges with volatility surging to between 31%-36% usually at the beginning or close of the year. NSE was relatively quiet except for mid-2016 when volatility surged as a result of the market fall. BSI and KSE show similar evolution of volatility over the sample period. It is not clear why this is so given that Kenya's stock market has largely service industries and BSI have commodity firms.

Seasonal effects due mainly to budget cycles and/or market timing could show up in the stock market as increased or decreased activity. Again, the observed thin and asynchronous trading during part of the year may be observed as calm or volatile sessions in the exchanges. To investigate these effects, we ranked the average monthly volatility across the sample period 2011 to 2017. We plotted to see how these activities affect the outcomes month-on-month volatility. The plot is shown in Figure 41-44.

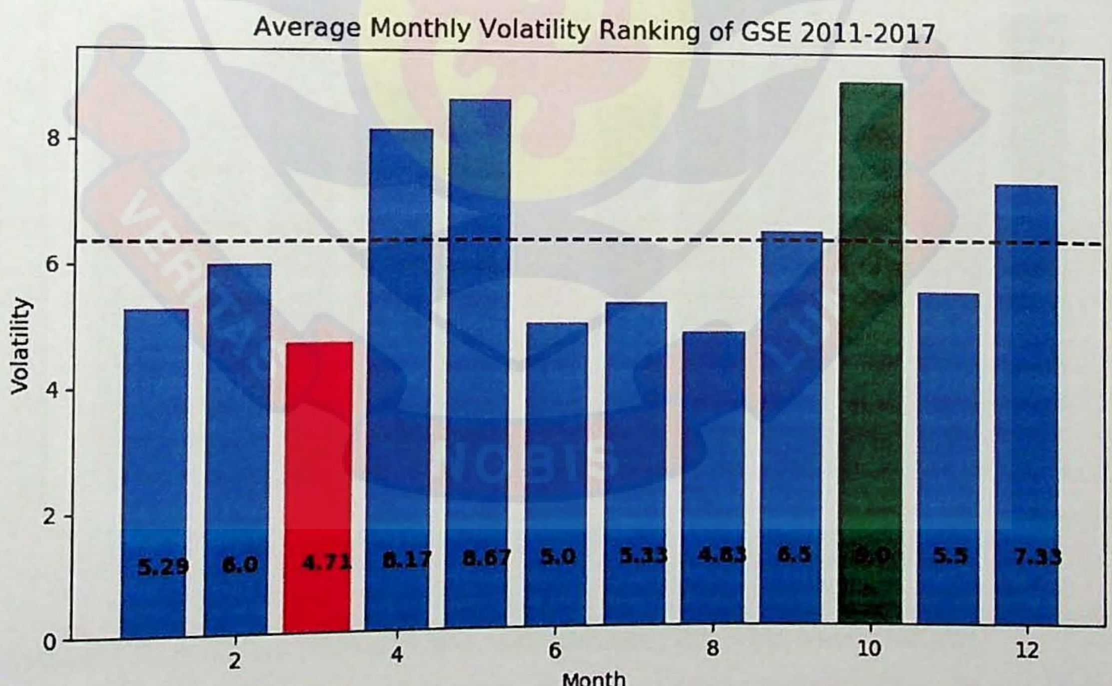

Figure 41: Average monthly volatility ranking for GSE Index returns

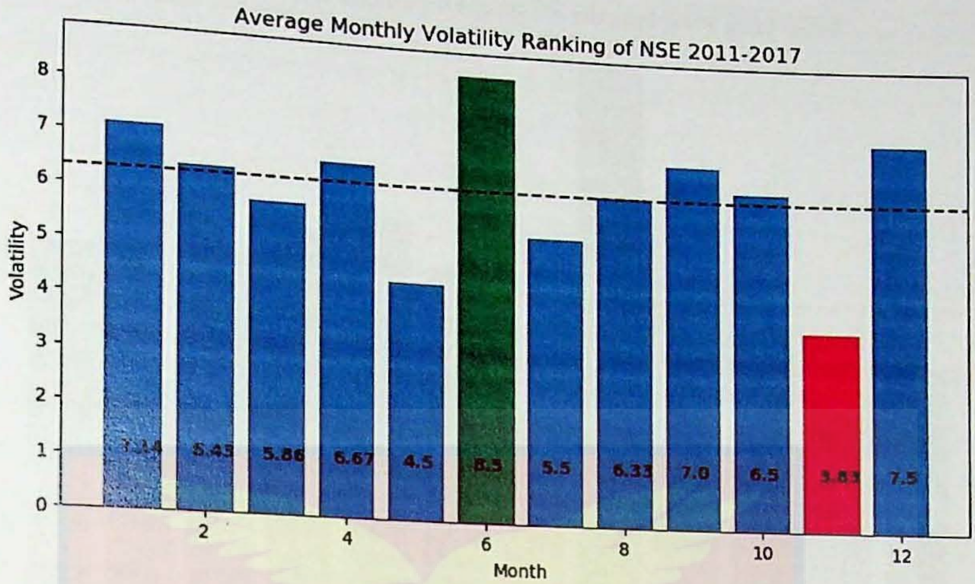

Figure 42: Average monthly volatility ranking for NSE Index returns

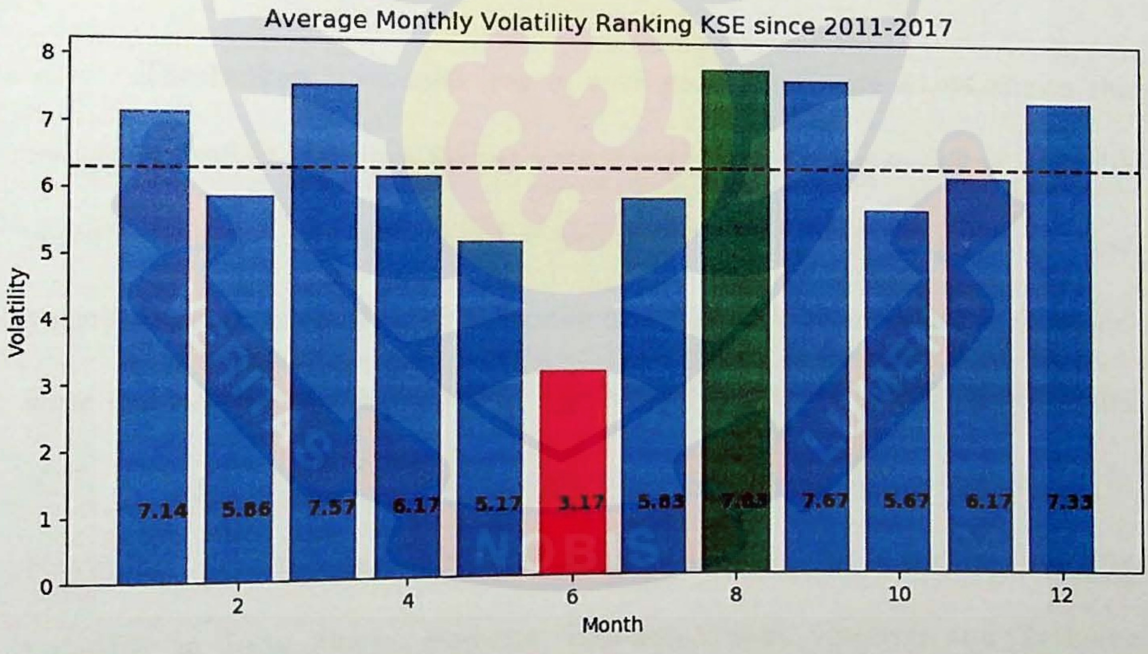

Figure 43: Average monthly volatility ranking for KSE Index returns

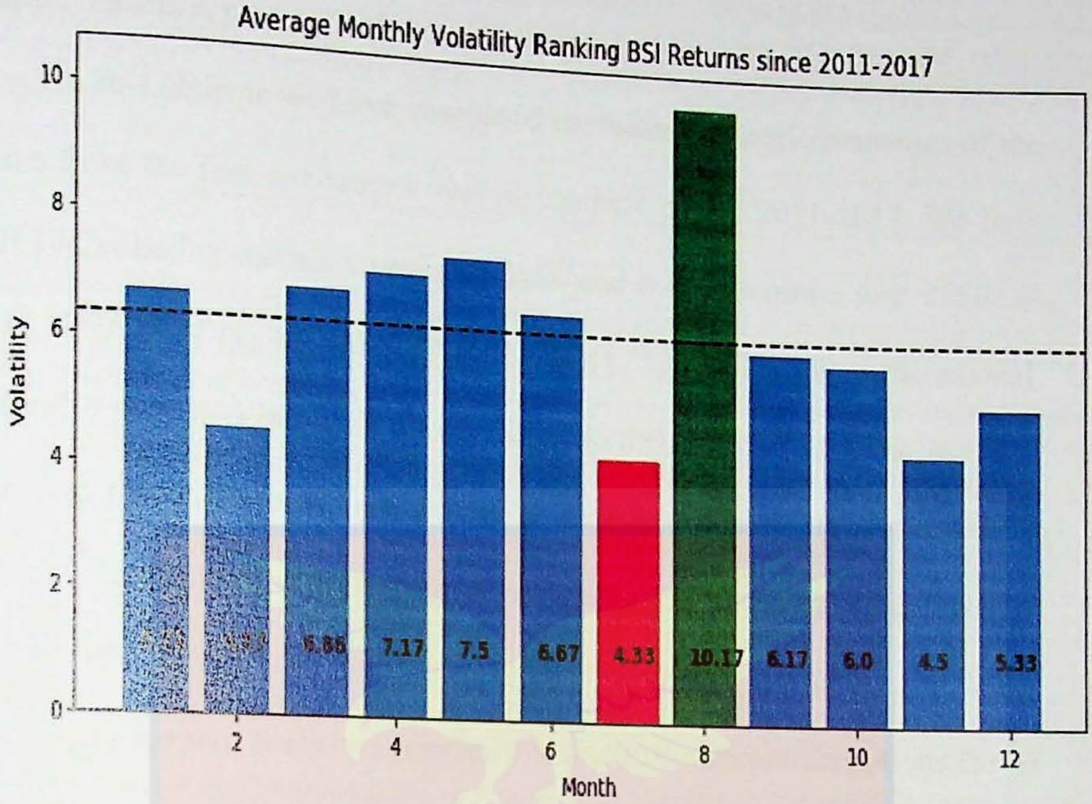

Figure 44: Average monthly volatility ranking for BSI Index returns

The broken horizontal line in each graph in Figure 41-44 shows the average monthly volatility for the exchanges over the sample period with the months from January to December numbered 1 to 12. GSE experiences low volatility on average in the first quarter and the markets get giddy in October. June and August are the low and high volatility months respectively for the KSE. In Nigeria, the NSE shows high volatility on June and low volatility in November. On the other hand, BSI's high volatility is in August and its low volatility in July. Zhang, Sornette, Balcilar, Gupta, Ozdemir and Yetkiner (2016) have documented crises in financial markets over time and there is indication that October is the most volatile months of all for much of the developed and emerging markets. This apparent contrast could explain the low correlation between stock market returns between developed and frontier markets.

Chapter Summary

In this chapter we have examined the heteroscedastic outcomes of the returns from the four exchanges over our sample period 2011-2017. We built in all 144 volatility models covering single- and double-regimes with GARCH (1, 1), EGARCH (1, 1) and GJR-GARCH (1, 1) models using the normal, Student's *t* and GED innovations together with their skewed versions. We then resampled the data from daily returns to monthly returns and examined the least and most volatile months across the sample period. Further, we looked at the correlations across the markets and as well as the tails of the returns. In the next chapter we will draw our conclusions and make recommendations for all stakeholders and for further study.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Overview

We present in this work evidence of the presence of regime switching in the conditional volatility of returns in the equity markets of some sub-Saharan African countries. The findings, conclusions and recommendations are set out in this chapter.

Summary

Risk estimation in frontier markets present a lot of challenges in the finance discipline. For the equity markets in sub-Saharan African countries, this problem becomes even harder as a result of changing background signals such as government economic and monetary policies, socio-political developments, information availability issues and financial reporting transparency all feeding into the DGP. This gives rise to the market regimes which we considered in investigating the changing levels of volatility in the sub-region equity markets. We addressed the evidence of regime switching against single regime GARCH models in the equity markets of Ghana, Nigeria, Kenya and Botswana. Again, switching behaviour gives rise to heavy-tails in the distribution of returns. The distributions in the left tails are leptokurtic and present hidden risks to investors and ultimately to risk management in trading and investment. An assessment of the nature of tails and their behaviour in this context was investigated.

The equities in our sampled markets exhibit heavy-tails with regime changes in volatility. The low stable volatility regime is the dominant one in all the markets. We found that the regime switching models describing the returns from these markets are heterogeneous and market specific. The GSE admits a two-regime EGARCH (1, 1) with skewed GED innovations implying the market's reaction to negative shocks is more pronounced than positive shocks and with heavy-tails. The asymmetry is the result of mainly leverage effects in the Ghanaian bourse. A two-regime GARCH (1, 1) estimates with GED innovations describes the evolution of volatility for the KSE index returns during the sample period. Volatility clustering characterises the returns and not much by way of leverage or information effects are present in the market. The two-regime GJR-GARCH (1, 1) estimates with skewed Student-*t* innovations for the NSE index returns offer a richer explanation of the behaviour of the volatility of returns in that market. Volatility here is influenced by the leverage effects which become the main determining factors once equity prices begin to fall. The rapid switching in volatility in the smoothed probability plot of Figure 20 is reflected in the GARCH (1, 1) with skewed Student-*t* estimates for the BSI index returns. Each of the regimes lasts only for very short periods on the BSI with no leverage effects in the market.

Conclusions

Stock markets are the pulse of the economy. They respond to the economy by picking signals in the background. They do not exist in a vacuum. Thus, any attempt at modelling the dynamics of stock markets particularly with respect to the embedded risks must take congnisance of the underlying signals.

Accurately forecasting volatility is important as the metric is a starting point in finance for pricing and risk management decisions. In risk management, consideration of what models to adopt for a particular market is purely an empirical question. And as we see in our analysis, the equity markets across sub-Saharan Africa are a disparate lot whose indices respond to various models. A search for volatility models in current finance literature across the sub-Saharan markets point to the pervasiveness of vanilla GARCH (1,1) non-switching models (Maqsood, Safdar, Shafi, & Lelit, 2017; Owidi & Mugo-Waweru, 2016; Omari-Sasu, Frempong, Boateng, & Boadi, 2015; Lesotho, Motlaleng, & Ntsosa, 2016; Osazevbaru, 2014; Olowe, 2009). These models underestimate the risks embedded in the data leading to under- or over-pricing of equity-related financial assets. The phenomenon can lead to inappropriate allocation of capital apart from giving investing firms and policymakers a false sense of security. Specifying the correct models require an understanding of the specific market environment – the socio-politico economic environment generating the data.

Regime switching volatility models offer a clearer insight about the risks embedded in the market data period to period and how investors, traders and policymakers can react. Classical GARCH models are insufficient in describing the changing volatility in the sub-Saharan African region. The work shows that incorporating regime changes in our models is the appropriate and most consistent way of estimating the parameters of the volatility of returns based on the underlying DGP. The classical GARCH models can be seen as basic artefacts for building realistic heteroscedastic functions that are in agreement with the data and at the same time reliable for investors and policymakers. The regime switching models aligned with works of Guidolin (2011) and Tronzano (2001) who seek to build models that adhere to the behaviour of the data such as monetary policy changes, political developments and economic turmoil. We thus see that with regime-switching, our models are brought closer to the data coming out of the equity markets. This aligns with the adjustment principle put forward by Kocherlakota (2007) in the economic policy sense of models mimicking closely the behaviour of the data.

Event over the past two decades point to increasingly unstable equity markets and as the markets become interconnected, contagion can spread very quickly. Thus, volatility modelling should take into consideration the environment of the market in its analysis. Risks are embedded in market data and data spring from environmental factors. Since market data is fundamental to trading and considerations involving risk management, regime changes should be among the central factors in volatility modelling.

The assumptions made about the DGP are what inform the choice of the model used in modelling the volatility. In the case of regime switching, models give a granular result of estimating volatility accurately for decisions as varied as portfolio construction, pricing of financial assets and risk management in general. For a long time now, problems of model misspecification have been the Achilles' heel of risk management in the financial industry. Simpson (2010) examined and lays bare this problem as contributing to the underestimation of risk in the lead up to the global financial crisis in 2008. This was a re-echo of the work of Derman (1996) who mentioned the importance of modeling assumptions in building models, emphasizing the need to let these models reflect as much as possible the reality in the financial markets.

It is noticeable that all the returns of the exchanges exhibit fatter tails although to various degrees, with the NSE's degree of kurtosis more severe than the rest. All these points at markets more prone to extreme return outcomes.

Financial crisis, political upheavals and social unrest, changing regulations and economic experimentation with fiscal and monetary policy all create uncertainties in the investment climate prompting a rethink on the part of the investor. A rise in volatility is associated with a slide downwards of equity prices, destroying the wealth of investors on the market (Crotty, 2009). In instances where there is prolonged volatility, Voth (2003) argues that the market will likely drag the whole economy through cascading effects on other asset classes like bonds, currencies and commodities into recession. This therefore has serious implications for policymaking. Risk provides opportunities for investments (Sharp, 1991). But as Shleifer and Vishny (2011) observed, increased and persistent volatility leads to a flight to safety of sovereign assets, leaving all other assets at the mercy of fire sales and further exacerbating the fires of uncertainty in the markets.

Recommendations

Investor complaint about the heightened risk of investing in sub-Saharan equities is not new (Zaremba & Maydybura, 2019; Chan-Lau, 2014). Indeed, risk in frontier markets is as it should be – always a feature of the market. These are markets yet to be integrated into global finance; hence there are pockets of hidden information that will likely trip both investors and policymakers alike. What we have been able to show is that regime switching models are the most appropriate and consistent with the data modelling the risk of markets returns. Given the background that leads to trading in the regions' bourses, we have shown that heavy-tailed regime switching models best fit the returns data of the markets. Trading and investment decisions will rely on these findings to implement price filters to minimise the risk of investing while increasing returns. This is an important contribution to the practice of investment and risk management in the information starved sub-Saharan Africa equity markets. Again, policy crafters at the central banks and relevant ministries will be alert on when the equity markets are in distress to calibrate interest rates and spending aimed at mitigating the disruption caused by the high volatility regimes.

This work adopted a reduced model approach in studying the regime switching behaviour of volatility of the equity markets of the sub-Saharan African markets. What we set out to investigate was: Do the underlying disturbances show up in the data by way of the changing levels of heteroscedasticity of market returns especially related to stocks? Based on a purely data-centric approach, this work has been able to demonstrate the existence of market regimes in the data without seeking to know what caused such regime switches and at what dates did they occur. Investors will like to know the answers to these questions in order to be guided going forward. Again, it will be interesting in a future study to see how regime switching influences VaR and the expected tail loss over time. This will be useful for policy in determining cyclically the capital adequacy requirements of financial institutions. A longer sample than was used in this work is required.

REFERENCES

Abad, P., Benito, S., & López, C. (2014). A comprehensive review of Value at Risk methodologies. *The Spanish Review of Financial Economics, 12*(1), 15-32.

Abdullahi, S. I. (2017). Stock Market Linkage, Financial Contagion and Assets Price Movements: Evidence from Nigerian Stock Exchange. *Journal of Advanced Studies in Finance, 8*(16), 146-159.

Abramsom, A., & Cohen, I. (2007). On the Stationarity of Markov-Switching GARCH Processes. *Econometric Theory, 23*, 485-500.

Adamu, A. (2009). The effects of global financial crisis on Nigerian economy. *International Journal of Investment and Finance, 1*, 11-21.

Adjasi, C. K. (2009). Macroeconomic uncertainty and conditional stock-price volatility in frontier African markets: Evidence from Ghana. *The Journal of Risk Finance, 10*(4), 333-349.

Afuecheta, E., Utazi, C., Ranganai, E., & Nnanatu, C. (2020). An Application of Extreme Value Theory for Measuring Financial Risk in BRICS Economies. *Annals of Data Science*, 1-40.

Aı, Y., & Kimmel, R. (2007). Maximum likelihood estimation of stochastic volatility models. *Journal of Financial Economics, 83*(2), 413-452.

Akella, S. R., & Chen, S. J. (1990). Interest rate sensitivity of bank stock returns: Specification effects and structural changes. *Journal of Financial Research, 13*(2), 147-154.

Alba, J. D., & Wang, P. (2017). Taylor rule and discretionary regimes in the United States: Evidence from a k-state Markov regime-switching model. *Macroeconomic Dynamics, 21*(3), 817-833.

Alchian, A. (1950). Uncertainty, Evolution and Economic Theory. *Journal of Political Economy, 58*, 211-221.

Alexander, C. (2008). *Market risk analysis, practical financial econometrics (Vol. 2)*. John Wiley & Sons.

Allington, N. F., McCombie, J. S., & Pike, M. (2012). Lessons not learned: from the collapse of Long-Term Capital Management to the subprime crisis. *Journal of Post Keynesian Economics, 34*(4), 555-582.

Aloui, C., & ben Hamida, H. (2014). Modelling and forecasting value at risk and expected shortfall for GCC stock markets: Do long memory, structural breaks, asymmetry, and fat-tails matter? *The North American Journal of Economics and Finance, 29*, 349-380.

Aloui, C., Hammoudeh, S., & Hamida, H. B. (2015). Price discovery and regime shift behavior in the relationship between sharia stocks and sukuk: A two-state Markov switching analysis. *Pacific-Basin Finance Journal, 34*, 121-135.

Andrle, M., Berg, A., Morales, R. A., Portillo, R., & Vlcek, J. (2015). On the Sources of Inflation in K enya: A Model-Based Approach. *South African Journal of Economics, 83*(4), 475-505.

Ang, A., & Bekaert, G. (2004). How regimes affect asset allocation. *Financial Analysts Journal, 60*(2), 86-99.

Ang, A., & Timmermann, A. (2012). Regime changes and financial markets. *Annu. Rev. Financ. Econ, 4*(1), 313-337.

Ardia, D., & Hoogerheide, L. F. (2010). Bayesian estimation of the garch (1, 1) model with student-t innovations. *The R Journal, 2*(2), 41-47.

Ardia, D., Bluteau, K., Boudt, K., & Catania, L. (2018). Forecasting Risk with Markov-Switching GARCH Models: A Large-Scale Performance Study. *International Journal of Forecasting, 34*(4), 733-747.

Ardia, D., Bluteau, K., Boudt, K., Catania, L., & Trottier, D. A. (2019). Markov-switching GARCH models in R: The MSGARCH package. *Journal of Statistical Software, 91*(4), 1-38. doi:10.18637/jss.v091.i04

Assaf, A. (2009). Extreme observations and risk assessment in the equity markets of MENA region: Tail measures and Value-at-Risk. *International Review of Financial Analysis, 18*(3), 109-116.

Assaf, A. (2015). Value-at-Risk analysis in the MENA equity markets: Fat tails and conditional asymmetries in return distributions. *Journal of Multinational Financial Management, 29*, 30-45.

Assoe, K. (1998). Regime-switching in emerging stock market returns. *Multinational Journal, 2*, 101-132.

Atoi, N. V. (2014). Testing volatility in Nigeria stock market using GARCH models. *CBN Journal of Applied Statistics, 5*(2), 65-93.

Augustyniak, M. (2014). Maximum likelihood estimation of the Markov-switching GARCH model. *Computational Statistics & Data Analysis, 76*(1), 61–75.

Babikir, A., Gupta, R., Mwabutwa, C., & Owusu-Sekyere, E. (2012). Structural breaks and GARCH models of stock return volatility: The case of South Africa. *Economic Modelling, 29*(6), 2435-2443.

Bahlous, M. (2013). Does Cross-Listing Benefit the Shareholders? Evidence from Companies in the GCC Countries? *Asia-Pacific Financial Markets, 20*(4), 345-381.

Bai, J., & Perron, P. (1998). Estimating and testing linear models with multiple structural changes . *Econometrica*, 47-78.

Bai, J., & Perron, P. (2003a). Computation and analysis of multiple structural change models. *Journal of Applied Econometrics, 18* (1), 1-22.

Bai, J., & Perron, P. (2003b). Critical values for multiple structural change tests. *The Econometrics Journal, 6*(1), 72-78.

Balcilar, M., Demirer, R., & Hammoudeh, S. (2013). Investor herds and regime-switching: Evidence from Gulf Arab stock markets. *Journal of International Financial Markets, Institutions and Money, 23*, 295-321.

Barclays. (2018). *Investing in frontier markets*. Retrieved from https://www.barclays.co.uk/smart-investor/news-and-research/investment-strategies/investing-in-frontier-markets/

Barndorff-Nielsen, O., & Shephard, N. (2004). Power and bipower variation with stochastic volatility and jumps (with discussion). *Journal of Financial Econometrics, 2*, 1-48.

Barndorff-Nielsen, O., & Shephard, N. (2006). Variation, jumps, and high frequency data in financial econometrics. In R. Blundell, P. Torsten, & W. Newey, *Advances in Economics and Econometrics: Theory and Applications.* Cambridge: Cambridge University Press.

Barry, D., & Hartigan, J. A. (1992). Product partition models for change point problems. *Annals of Statistics, 20*, 260–279.

Barry, D., & Hartigan, J. A. (1993). A Bayesian analysis for change point problems. *Journal of the American Statistical Association, 88*(421), 309-319.

Basel Committee on Banking Supervision. (2011). *Revisions to the Basel II Market Risk Framework.* Bank for International Settlements.

Baur, D. G., & Glover, K. J. (2014). Heterogeneous expectations in the gold market: Specification and estimation. *Journal of Economic Dynamics and Control, 40*, 116-133.

Bauwens, L., Preminger, A., & Rombouts, J. V. (2010). Theory and inference for a Markov switching GARCH model. *The Econometrics Journal, 13*(2), 218-244.

Beirlant, J., Vynckier, P., & Teugels, J. L. (1996). Tail index estimation, pareto quantile plots regression diagnostics. *Journal of the American Statistical Association, 91*(436), 1659-1667.

Bekaert, G., Erb, C. B., Harvey, C. R., & Viskanta, T. E. (1998). Distributional characteristics of emerging market returns and asset allocation. *Journal of Portfolio Management, 24*(2), 102-116.

Ben-David, D., & Papell, D. H. (1995). The great wars, the great crash, and steady state growth: Some new evidence about an old stylized fact. *Journal of Monetary Economics, 36*(3), 453-475.

Berger, D., Pukthuanthong, K., & Yang, J. J. (2011). International diversification with frontier markets. *Journal of Financial Economics, 101*(1), 227-242.

Berger, J. O., Bernardo, J. M., & Sun, D. (2015). Overall objective priors. *Bayesian Analysis, 10*(1), 189-221.

Berkowitz, J., Christoffersen, P., & Pelletier, D. (2011). Evaluating value-at-risk models with desk-level data. *Management Science, 57*(12), 2213-2227.

Bernardi, M., & Catania, L. (2016). Comparison of Value–at–Risk models using the MCS approach. *Computational Statistics, 31*(2), 579–608. doi:https://doi.org/10.1007/s00180-016-0646-6

Białkowski, J., Bohl, M. T., Stephan, P. M., & Wisniewski, T. P. (2015). The gold price in times of crisis. *International Review of Financial Analysis, 41*, 329-339.

Billio, M., & Cavicchioli, M. (2017). Markov Switching GARCH Models: Filtering, Approximations and Duality. In *Mathematical and Statistical Methods for Actuarial Sciences and Finance* (pp. 59-72). Springer, Cham.

Billio, M., Casarin, R., & Osuntuyi, A. (2016). Efficient Gibbs sampling for Markov switching GARCH models. *Computational Statistics & Data Analysis, 100*, 37-57.

Bjørnland, H. C., & Leitemo, K. (2009). Identifying the interdependence between US monetary policy and the stock market. *Journal of Monetary Economics, 56*(2), 275-282.

Black, F. (1976). Studies of Stock Price Volatility Changes. *Proceedings of the 1976 Meetings of the Business and Economics Statistics Section* (pp. 177-181). American Statistical Association.

Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy, 81*(3), 637-654.

Black, F., & Scholes, M. (1973). The Pricing of Optionsand Corporate Liabilities. *Journal of Political Economy, 81*, 637-654.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics, 31*(3), 307-327.

Bolokwe, K., & Sedimo, K. (2019). Botswana. In H. Raubenheimer, *African Capital Markets Challenges and Opportunities* (pp. 13-21). CFA Institute Research Foundation.

Bonilla, C. A., & Sepulveda, J. (2011). Stock returns in emerging markets and the use of GARCH models. *Applied Economics Letters, 18*(14), 1321-1325.

Bosworth, B. P., Collins, S. M., & Reinhart, C. M. (1999). Capital flows to developing economies: implications for saving and investment. *Brookings papers on economic activity, 1999*(1), 143-180.

Box, G. E., & Pierce, D. A. (1970). Distribution of residual autocorrelations in autoregressive-integrated moving average time series models. *Journal of the American statistical Association, 65*(332), 1509-1526.

Busetti, F., & Taylor, A. R. (2003). Variance shifts, structural breaks, and stationarity tests. *Journal of Business & Economic Statistics, 21*(4), 510-531.

Busse, M., & Hefeker, C. (2007). Political risk, institutions and foreign direct investment. *European Journal of Political Economy, 23* (2), 397-415.

Cai, J. (1994). A Markov model of switching-regime ARCH. *Journal of Business & Economic Statistics, 12* (3), 309-316.

Campbell, S. (2005). *A review of backtesting and backtesting procedures (Discussion Paper No. 2005-21).* Washington, D.C: Federal Reserve Board.

Carlin, B., Gelfand, A., & Smith, A. F. (1992). Hierarchical Bayesian analysis of change-point problems. *Applied Statistics, 41*, 389–405.

Carrasco, M. (2002). Misspecified structural change, threshold, and Markov-switching models. *Journal of Econometrics, 109*(2), 239-273.

Carrion-i-Silvestre, J. L., & Sansó, A. (2006). A guide to the computation of stationarity tests . *Empirical Economics, 31*(2), 433-448.

Carsamer, E. (2016). Volatility Transmission In Selected African Foreign Exchange Markets. *International Journal of Economic Perspectives, 10*(2), 43-61.

Cerović Smolović, J., Lipovina-Božović, M., & Vujošević, S. (2017). GARCH models in value at risk estimation: empirical evidence from the Montenegrin stock exchange. *Economic research-Ekonomska istraživanja, 30*(1), 477-498.

Chan-Lau, J. A. (2014). Frontier equity markets: risk parity lessons for asset allocation. *The Journal of Alternative Investments, 16*(4), 28-36.

Charfeddine, L., & Ajmi, A. N. (2013). The Tunisian stock market index volatility: Long memory vs. switching regime. *Emerging Markets Review, 16*, 170-182.

Chen, M. P., Chen, P. F., & Lee, C. C. (2014). Frontier stock market integration and the global financial crisis. *The North American Journal of Economics and Finance, 29*, 84-103.

Cheng, P. K. (2020). Listen to the signals from an interactive agent-based model. *Applied Economics Letters*, 1-5.

Chib, S., & Greenberg, E. (1995). Understanding the metropolis-hastings algorithm. *The American Statistician, 49*(4), 327-335.

Chib, S., & Greenberg, E. (1995). Understanding the metropolis-hastings algorithm. *The American Statistician, 49*(4), 327-335.

Chincarini, L. B. (2007). The Amaranth Debacle: A Failure of Risk Measures or a Failure of Risk Management? *The Journal of Alternative Investments, 10*(3), 91-104.

Chinzara, Z., & Slyper, S. (2013). Volatility and anomalies in the Johannesburg Securities Exchange daily returns. *Financial Markets Journal, 17* (1), 25-41.

Chkili, W., & Nguyen, D. K. (2014). Exchange rate movements and stock market returns in a regime-switching environment: Evidence for BRICS countries. *Research in International Business and Finance, 31*, 46-56.

Christie, A. A. (1982). The stochastic behavior of common stock variances: Value, leverage and interest rate effects. *Journal of Financial Economics, 10*(4), 407-432.

Christoffersen, P. F. (1998). Evaluating interval forecasts. *International Economic Review*, 841-862.

Chuffart, T. (2015). Selection criteria in regime switching conditional volatility models. *Econometrics, 3*(2), 289-316.

Claeys, P. (2015). Timing and duration of inflation targeting regimes. *Ensayos sobre Política Económica, 33*(76), 18-30.

Clyde, M., & George, E. I. (2004). Model uncertainty . *Statistical science*, 81-94.

Cont, R. (2001). Empirical properties of asset returns: stylized facts and statistical issues. *Quantitative Finance, 1*(2), 223-236. doi:https://doi.org/10.1080/713665670

Cont, R. (2007). Volatility clustering in financial markets: empirical facts and agent-based models. In *Long memory in economics* (pp. 289-309). Berlin, Heidelberg: Springer.

Cowles, M. K., & Carlin, B. P. (1996). Markov Chain Monte Carlo Convergence Diagnostics: A Comparative Review. *Journal of the American Statistical Association, 91*(434), 883-904.

Creal, D., Koopman, S. J., & Lucas, A. (2013). Generalized autoregressive score models with applications. *Journal of Applied Econometrics, 28*(5), 777–795. doi:https://doi.org/10.1002/jae.1279

Crotty, J. (2009). *The bonus-driven "rainmaker" financial firm: How these firms enrich top employees, destroy shareholder value and create system financial instability (No. 2009-13).* Working Paper.

Curto, J. D., Pinto, J. C., & Tavares, G. N. (2009). Modeling stock markets' volatility using GARCH models with Normal, Student's t and stable Paretian distributions. *Statistical Papers, 50*(2), 311-330.

Daal, E., Naka, A., & Yu, J. S. (2007). Volatility clustering, leverage effects, and jump dynamics in the US and emerging Asian equity markets. *Journal of Banking & Finance, 31*(9), 2751-2769.

Danıelsson, J. (2002). The emperor has no clothes: Limits to risk modelling. *Journal of Banking & Finance, 26*(7), 1273-1296.

Danielsson, J. (2008). Blame the models. *Journal of Financial Stability, 4*(4), 321-328.

Darné, O., & Diebolt, C. (2004). Unit roots and infrequent large shocks: New international evidence on output . *Journal of Monetary Economics, 51*(7), 1449-1465.

Das, D., & Yoo, B. H. (2004). A Bayesian MCMC algorithm for Markov switching GARCH models. *Econometric Society*, 1-16.

Davig, T. (2004). Regime-switching debt and taxation. *Journal of Monetary Economics, 51*(4), 837-859.

Della Croce, R., Stewart, F., & Yermo, J. (2011). Promoting longer-term investment by institutional investors. *OECD Journal: Financial Market Trends, 2011*(1), 145-164.

Derman, E. (1996). *Model risk.* Goldman Sachs Quantitative Strategies Research Notes.

Dickson, J. (2013, 12 01). *http://www.ashmore.com*. Retrieved from The Emerging View: http://www.ashmoregroup.com/sites/default/files/article-docs/2013%2012%20F%20Frontier%20Equities_0.pdf

Diebold, F. X., & Inoue, A. (2001). Long memory and regime switching. *Journal of econometrics, 105*(1), 131-159.

Diebold, F. X., Lee, J. H., & Weinbach, G. C. (1994). Regime switching with time-varying transition probabilities. *Business Cycles: Durations, Dynamics, and Forecasting, 1*, 144-165.

Dimic, N., Orlov, V., & Piljak, V. (2015). The political risk factor in emerging, frontier, and developed stock markets. *Finance Research Letters, 15*, 239-245.

Diongue, A. K., Guegan, D., & Wolff, R. C. (2010). BL-GARCH models with elliptical distributed innovations. *Journal of Statistical Computation and Simulation, 80*(7), 775-791.

Duffy, J., & Engle-Warnick, J. (2006). Multiple regimes in US monetary policy? A nonparametric approach . *Journal of Money, Credit and Banking*, 1363-1377.

Ederington, L. H., & Guan, W. (2006). Measuring historical volatility. *Journal of Applied Finance, 16*(1), 5-14.

Eichengreen, B., & Bordo, M. D. (2002). *Crises now and then: What lessons from the last era of financial globalization (No. w8716).* National Bureau of Economic Research.

Elyasiani, E., & Mansur, I. (1998). Sensitivity of the bank stock returns distribution to changes in the level and volatility of interest rate: A GARCH-M model. *Journal of Banking & Finance, 22*(5), 535-563.

Embrechts, P., Klüppelberg, C., & Mikosch, T. (2013). *Modelling extremal events: for insurance and finance (Vol. 33).* Springer Science & Business Media.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica: Journal of the Econometric Society*, 987-1007.

Engle, R. F. (2001). GARCH 101: The use of ARCH/GARCH models in applied econometrics. *Journal of Economic Perspectives, 15*(4), 157-168.

Engle, R. F. (2004). Risk and volatility: Econometric models and financial practice. *American Economic Review, 94* (3), 405-420.

Engle, R. F., & Manganelli, S. (2004). CAViaR: Conditional autoregressive value at risk by regression quantiles. *Journal of Business & Economic Statistics, 22*(4), 367-381.

Erdman, C., & Emerson, J. W. (2007). bcp: an R package for performing a Bayesian analysis of change point problems. *Journal of Statistical Software, 23* (3), 1-13.

Esman Nyamongo, M., & Misati, R. (2010). Modelling the time-varying volatility of equities returns in Kenya. *African Journal of Economic and Management Studies, 1*(2), 183-196.

Fama, E. (1963). 'Mandelbrot and the stable partition hypothesis. *Journal of Business, 36*(4), 420–429.

Fan, Y., Zhang, Y. J., Tsai, H. T., & Wei, Y. M. (2008). Estimating 'Value at Risk'of crude oil price and its spillover effect using the GED-GARCH approach. *Energy Economics, 30*(6), 3156-3171.

Feuerverger, A., & Hall, P. (1999). Estimating a tail exponent by modelling departure from a Pareto distribution. *The Annals of Statistics, 27*(2), 760-781.

Fiess, N., & Shankar, R. (2009). Determinants of exchange rate regime switching. *Journal of International Money and Finance, 28*(1), 68-98.

Fisher, R., & Tippett, L. (1928). Limiting forms of the frequency distribution of the largest and smallest member of a sample. *Proc. Camb. Phil. Soc., 24*(2), 180–190. doi:10.1017/s0305004100015681

Elyasiani, E., & Mansur, I. (1998). Sensitivity of the bank stock returns distribution to changes in the level and volatility of interest rate: A GARCH-M model. *Journal of Banking & Finance, 22*(5), 535-563.

Embrechts, P., Klüppelberg, C., & Mikosch, T. (2013). *Modelling extremal events: for insurance and finance (Vol. 33)*. Springer Science & Business Media.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica: Journal of the Econometric Society*, 987-1007.

Engle, R. F. (2001). GARCH 101: The use of ARCH/GARCH models in applied econometrics. *Journal of Economic Perspectives, 15*(4), 157-168.

Engle, R. F. (2004). Risk and volatility: Econometric models and financial practice. *American Economic Review, 94* (3), 405-420.

Engle, R. F., & Manganelli, S. (2004). CAViaR: Conditional autoregressive value at risk by regression quantiles. *Journal of Business & Economic Statistics, 22*(4), 367-381.

Erdman, C., & Emerson, J. W. (2007). bcp: an R package for performing a Bayesian analysis of change point problems. *Journal of Statistical Software, 23* (3), 1-13.

Esman Nyamongo, M., & Misati, R. (2010). Modelling the time-varying volatility of equities returns in Kenya. *African Journal of Economic and Management Studies, 1*(2), 183-196.

Fama, E. (1963). 'Mandelbrot and the stable partition hypothesis. *Journal of Business, 36*(4), 420–429.

Fan, Y., Zhang, Y. J., Tsai, H. T., & Wei, Y. M. (2008). Estimating 'Value at Risk'of crude oil price and its spillover effect using the GED-GARCH approach. *Energy Economics, 30*(6), 3156-3171.

Feuerverger, A., & Hall, P. (1999). Estimating a tail exponent by modelling departure from a Pareto distribution. *The Annals of Statistics, 27*(2), 760-781.

Fiess, N., & Shankar, R. (2009). Determinants of exchange rate regime switching. *Journal of International Money and Finance, 28*(1), 68-98.

Fisher, R., & Tippett, L. (1928). Limiting forms of the frequency distribution of the largest and smallest member of a sample. *Proc. Camb. Phil. Soc., 24*(2), 180–190. doi:10.1017/s0305004100015681

Franses, P. H., & Van Dijk, D. (1996). Forecasting stock market volatility using (non-linear) Garch models. *Journal of Forecasting, 15*(3), 229-235.

FTSE. (2019). *FTSE Frontier Index Series.* Retrieved 08 2019, from https://research.ftserussell.com/products/downloads/FTSE_Frontier_Index_Series_Ground_Rules.pdf

FTSE-Russell. (2019, 12 1). *FTSE equity country classification process.* Retrieved from https://content.ftserussell.com/sites/default/files/research/ftse_country_classification_process_final.pdf

Furbush, D. (1989). Program trading and price movement: Evidence from the October 1987 market crash. *Financial Management*, 68-83.

Galeano, P., & Ausín, M. C. (2010). The gaussian mixture dynamic conditional correlation model: Parameter estimation, value at risk calculation, and portfolio selection. *Journal of Business & Economic Statistics, 28*(4), 559-571.

Gao, C. T., & Zhou, X. H. (2016). Forecasting VaR and ES using dynamic conditional score models and skew Student distribution. *Economic Modelling, 53*, 216-223. doi:https://doi.org/10.1016/j.econmod.2015.12.004

Gelman, A. (2006). Prior distributions for variance parameters in hierarchical models. *Bayesian analysis, 1*(3), 515-534.

Gelman, A. C., Stern, H. S., Dunson, D. B., Vehtari, A., & Rubin, D. B. (2013). *Bayesian Data Analysis.* Chapman and Hall/CRC.

Gelman, A., Simpson, D., & Betancourt, M. (2017). The prior can often only be understood in the context of the likelihood. *Entropy, 19*(10), 1-13.

Gençay, R., & Selçuk, F. (2004). Extreme value theory and Value-at-Risk: Relative performance in emerging markets. *International Journal of Forecasting, 20*(2), 287-303.

Geng, J. B., Ji, Q., & Fan, Y. (2016). The impact of the North American shale gas revolution on regional natural gas markets: Evidence from the regime-switching model. *Energy Policy, 96*, 167-178.

Geweke, J. (1992). Evaluating the accuracy of sampling-based approaches to the calculations of posterior moments. *Bayesian Statistics, 4*, 641-649.

Geweke, J., & Keane, M. (2001). Computationally intensive methods for integration in econometrics. In *Handbook of econometrics* (Vol. 5, pp. 3463-3568). Elsevier.

Ghemawat, P. (2001). Distance still matters. *Harvard Business Review, 79*(8), 137-147.

Giacomini, R., & Komunjer, I. (2005). Evaluation and Combination of Conditional Quantile Forecasts. *Journal of Business and Economic Statistics, 23*(4), 416-431.

Glasserman, P., Heidelberger, P., & Shahabuddin, P. (2002). Portfolio value-at-risk with heavy-tailed risk factors. *Mathematical Finance, 12*(3), 239-269.

Glosten, L., Jagannathan, R., & Runkle, D. (1993). On the Relation Between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. *Journal of Finance, 48*(5), 1779-1801. doi:10.1111/j.1540-6261.1993.tb05128.x

Gnedenko, B. (1943). Sur la distribution limite du terme maximum d'une serie aleatoire. *Annals of Mathematics, 44*(3), 423–453. doi:10.2307/1968974

Goldfeld, S. M., & Quandt, R. E. (1973). A Markov model for switching regressions. *Journal of Econometrics, 1*(1), 3-15.

Gray, J. B., & French, D. W. (1990). Empirical comparisons of distributional models for stock index returns. *Journal of Business Finance & Accounting, 17*(3), 451-459.

Gray, S. F. (1996). Modeling the conditional distribution of interest rates as a regime-switching process. *Journal of Financial Economics, 42*(1), 27-62.

Griffin, B., Lystra, M., & Quisenberry, J. C. (2015). *Navigating the Frontier Equity Markets. Russell Research.* Russell Investments.

Guidolin, M. (2011). Markov switching models in empirical finance. In *Advances in econometrics* (pp. 1-86).

Günay, S. (2017). Value at risk (VaR) analysis for fat tails and long memory in returns. *Eurasian Economic Review, 7*(2), 215-230.

Haas, M., Mittnik, S., & Paolella, M. S. (2004). A New Approach to Markov-Switching GARCH Models. *Journal of Financial Econometrics, 2*(4), 493-530.

Hamilton, J. D. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica: Journal of the Econometric Society*, 357-384.

Hamilton, J. D. (1990). Analysis of time series subject to changes in regime. *Journal of Econometrics, 45*(1-2), 39-70.

Hamilton, J. D. (2016). Macroeconomic regimes and regime shifts. In J. B. Taylor, & H. Uhlig, *Handbook of macroeconomics* (Vol. 2, pp. 163-201). Amsterdam, Holland: Elsevier.

Hamilton, J. D., & Susmel, R. (1994). Autoregressive conditional heteroskedasticity and changes in regime. *Journal of Econometrics, 64*(1-2), 307-333.

Hansen, P. R., & Lunde, A. (2005). A forecast comparison of volatility models: does anything beat a GARCH (1, 1)? *Journal of Applied Econometrics, 20*(7), 873-889.

Hardy, M. R. (2001). A regime-switching model of long-term stock returns. *North American Actuarial Journal, 5*(2), 41-53.

Harmantzis, F. C., Miao, L., & Chien, Y. (2006). Empirical study of value-at-risk and expected shortfall models with heavy tails. *The Journal of Risk Finance*, 117-135.

Harvey, A. C. (2013). *Dynamic Models for Volatility and Heavy Tails: With Applications to Financial and Economic Time Series.* Cambridge University Press.

Hastings, W. K. (1970). Monte Carlo sampling methods using Markov chains and their applications. *Biometrika, 57*, 97–109.

Hauser, S., Marcus, M., & Yaari, U. (1994). Investing in emerging stock markets: Is it worthwhile hedging foreign exchange risk?. Journal of Portfolio Management. *20*(3), 76-81.

Hesse, I. (2019). Ghana. In H. Raubenheimer, *African Capital Markets Challenges and Opportunities* (pp. 73-82). CFA Institute Research Foundation.

Hilal, S., Poon, S. H., & Tawn, J. (2011). Hedging the black swan: Conditional heteroskedasticity and tail dependence in S&P500 and VIX. *Journal of Banking & Finance, 35*(9), 2374-2387.

Hill, B. M. (1975). A simple general approach to inference about the tail of a distribution. *Annals of Mathematical Statistics, 3*, 1163-1174.

Hobijn, B., Franses, P. H., & Ooms, M. (2004). Generalizations of the KPSS-test for stationarity. *Statistica Neerlandica, 58*(4), 483-502.

Hyndman, R. J., & Athanasopoulos, G. (2018). *Forecasting: Principles and Practice.* OTexts.

Ibragimov, M., Ibragimov, R., & Kattuman, P. (2013). Emerging markets and heavy tails. *Journal of Banking & Finance, 37*(7), 2546-2559.

Janczura, J., & Weron, R. (2012). Efficient estimation of Markov regime-switching models: An application to electricity spot prices. *AStA Advances in Statistical Analysis, 96* (3), 385-407.

Jones, C. P., Walker, M. D., & Wilson, J. W. (2004). Analyzing Stock Market Volatility Using Extreme-Day Measures. *Journal of Financial Research, 27*(4), 585-601.

Karolyi, G. (2015). *Cracking the emerging markets enigma, Financial Management Association survey and synthesis series.* New York: Oxford University Press, Oxford.

Kass, R. E., & Wasserman, L. (1996). The selection of prior distributions by formal rules. *Journal of the American Statistical Association, 91*(435), 1343-1370.

Kawata, R., & Kijima, M. (2007). Value-at-risk in a market subject to regime switching. *Quantitative Finance, 7*(6), 609-619.

Kearney, C. (2012). Emerging markets research: Trends, issues and future directions. *Emerging Markets Review, 13*, 159–183.

Khalifa, A. A., Hammoudeh, S., & Otranto, E. (2014). Patterns of volatility transmissions within regime switching across GCC and global markets. *International Review of Economics & Finance, 29*, 512-524.

Kim, C. J., & Nelson, C. R. (1999). Has the US economy become more stable? A Bayesian approach based on a Markov-switching model of the business cycle. *Review of Economics and Statistics, 81*(4), 608-616.

Kim, S., Shephard, N., & Chib, S. (1998). Stochastic volatility: likelihood inference and comparison with ARCH models. *The Review of Economic Studies, 65*(3), 361-393.

Klaas, B. (2008). From miracle to nightmare: An institutional analysis of development failures in Côte d'Ivoire. *Africa Today, 55*(1), 109-126.

Klapper, L., Richmond, C., & Tran, T. (2013). *Civil conflict and firm performance: evidence from Cote d'Ivoire.* The World Bank.

Kleiber, C. (2018). Structural change in (economic) time series. In *Complexity and synergetics* (pp. 275-286). Springer, Cham.

Kocherlakota, N. R. (2007). Model fit and model selection - Review. *Federal Reserve Bank of Saint Louis, 89*(4), 349-360.

Koop, G., & Potter, S. M. (2007). Estimation and forecasting in models with multiple breaks. *The Review of Economic Studies, 74*(3), 763-789.

Kratz, M., & Resnick, S. I. (1996). The QQ-estimator and heavy tails. Stochastic Models. *12*(4), 699-724.

Kupiec, P. H. (1995). Techniques for Verifying the Accuracy of Risk Measurement Models. *The Journal of Derivatives, 3*(2), 73–84.

Kurov, A. (2010). Investor sentiment and the stock market's reaction to monetary policy. *Journal of Banking & Finance, 34*(1), 139-149.

Lamoureux, C. G., & Lastrapes, W. D. (1990). Persistence in variance, structural change, and the GARCH model. *Journal of Business & Economic Statistics, 8*(2), 225-234.

Lemoine, N. P. (2019). Moving beyond noninformative priors: why and how to choose weakly informative priors in Bayesian analyses. *Oikos*, 912-928.

Lesotho, O. K., Motlaleng, G. R., & Ntsosa, M. M. (2016). Stock market returns and exchange rates in Botswana. *African Journal of Economic Review, 4*(2), 16-42.

Lindström, E., & Regland, F. (2012). Modeling extreme dependence between European electricity markets. *Energy Economics, 34*(4), 899-904.

Ljung, G. M., & Box, G. E. (1978). On a measure of lack of fit in time series models. *Biometrika, 65*(2), 297-303.

Luo, C., Seco, L. A., Wang, H., & Dash Wu, D. (2010). Risk modeling in crude oil market: a comparison of Markov switching and GARCH models. *Kybernetes, 39*(5), 750-769.

Maheu, J. M., & Gordon, S. (2008). Learning, forecasting and structural breaks. *Journal of Applied Econometrics, 23*(5), 553-583.

Maqsood, A., Safdar, S., Shafi, R., & Lelit, N. J. (2017). Modeling stock market volatility using GARCH models: A case study of Nairobi Securities Exchange (NSE). *Open Journal of Statistics, 7*(2), 369-381.

Marcucci, J. (2005). Forecasting Stock Market Volatility with Regime-Switching GARCH Models. *Studies in Nonlinear Dynamics & Econometrics, 9*(4), -. doi:10.2202/1558-3708.1145

Marshall, B. R., Nguyen, N. H., & Visaltanachoti, N. (2013). Liquidity measurement in frontier markets. *Journal of International Financial Markets, Institutions and Money, 27*, 1-12.

McAleer, M., & Da Veiga, B. (2008). Single-index and portfolio models for forecasting value-at-risk thresholds. *Journal of Forecasting, 27* (3), 217-235.

McMillan, D. G., & Speight, A. E. (2004). Daily volatility forecasts: Reassessing the performance of GARCH models. *Journal of Forecasting, 23*(6), 449-460.

McNeil, A. J., & Frey, R. (2000). Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach. *Journal of Empirical Finance, 7*(3-4), 271-300.

Merton, C. R. (1974). On the Pricing of Corporate Debt:The Risk Structure of Interest Rates. *Journal of Finance, 29*, 449-470.

Metropolis, N., Rosenbluth, A., Rosenbluth, M., Teller, A., & Teller, E. (1953). Equation of state calculations by fast computing machines. *Journal of Chemical Physics, 21*, 1087–1092.

Miao, D. W., Wu, C. C., & Su, Y. K. (2013). Regime-switching in volatility and correlation structure using range-based models with Markov-switching. *Economic Modelling, 31*, 87-93.

Miles, W. (2005). Do frontier equity markets exhibit common trends and still provide diversification opportunities? *International Economic Journal, 19*(3), 473-482.

Mosley, L., & Singer, D. A. (2008). Taking stock seriously: Equity-market performance, government policy, and financial globalization. *International Studies Quarterly, 52*(2), 405-425.

MSCI Frontier Markets. (2019, May 16). Retrieved from Market Classification: https://www.msci.com/market-classification

Müller, P. (1993). A generic approach to posterior integration and Gibbs sampling. Purdue University, Department of Statistics.

Muralidharan, K., & Niehaus, P. (2017). Experimentation at scale . *Journal of Economic Perspectives, 31*(4), 103-124.

Nelson, D. B. (1991). Conditional Heteroskedasticity in Asset Returns: A New Approach. *Econometrica*, 347-370. doi:10.2307/2938260

Nortey, E. N., Asare, K., & Mettle, F. O. (2015). Extreme value modelling of Ghana stock exchange index. *SpringerPlus, 4*(1), 1-17.

Nunes, L. C., Newbold, P., & Kuan, C. M. (1997). Testing for unit roots with breaks: evidence on the great crash and the unit root hypothesis reconsidered. *Oxford Bulletin of Economics and Statistics, 59*(4), 435-448.

Odell, J., & Ali, U. (2016). ESG investing in emerging and frontier markets. *Journal of Applied Corporate Finance, 28*(2), 96-101.

Offoing, A. I., Riman, H. B., & Godwin, J. B. (2018). Financial Contagion and Its Impact on the Nigerian Stock Market. *Journal of Economics and Business, 1*(3), 268-281.

Olowe, R. A. (2009). Stock return volatility, global financial crisis and the monthly seasonal effect on the Nigerian stock exchange . *African Review of Money Finance and Banking*, 73-107.

Olowe, R. A. (2009). Stock return, volatility and the global financial crisis in an emerging market: The Nigerian case. *International Review of Business Research Papers, 5*(4), 426-447.

Olson, D. L., & Wu, D. (2013). The impact of distribution on value-at-risk measures. *Mathematical and Computer Modelling, 58*(9-10), 1670-1676.

Omari-Sasu, A. Y., Frempong, N. K., Boateng, M. A., & Boadi, R. K. (2015). Modeling Stock Market Volatility Using GARCH Approach on the Ghana Stock Exchange. *International Journal of Business and Management, 10*(11), 169-176.

Osazevbaru, H. O. (2014). Measuring Nigerian stock market volatility. *Singaporean Journal of Business, Economics and Management Studies, 51*(1122), 1-14.

Owidi, O. H., & Mugo-Waweru, F. (2016). Analysis of asymmetric and persistence in stock return volatility in the Nairobi securities exchange market phases. *Journal of Finance and Economics, 4*(3), 63-73.

Owyang, M. T., & Ramey, G. (2004). Regime switching and monetary policy measurement. *Journal of Monetary Economics, 51*(8), 1577-1597.

Perron, P. (1989). The great crash, the oil price shock, and the unit root hypothesis . *Econometrica: Journal of the Econometric Society*, 1361-1401.

Pickands III, J. (1975). Statistical inference using extreme order statistics. *Annals of Statistics, 3*(1), 119-131.

Piger, J. (2009). Econometrics: Models of Regime Changes. In R. Meyers, *Complex Systems in Finance and Econometrics.* New York, NY: Springer.

Pincus, S., & Kalman, R. E. (2004). Irregularity, volatility, risk, and financial market time series. *Proceedings of the National Academy of Sciences, 101*(38), 13709-13714.

Polson, N. G., & Scott, J. G. (2012). On the half-Cauchy prior for a global scale parameter. *Bayesian Analysis, 7*(4), 887-902.

Poon, S. H., & Granger, C. (2005). Practical issues in forecasting volatility. *Financial Analysts Journal, 61*(1), 45-56.

Quandt, R. E. (1958). The estimation of the parameters of a linear regression system obeying two separate regimes. *Journal of the American Statistical Association, 53*(284), 873-880.

Quisenberry Jr, C., & Griffith, B. (2010). Frontier equity markets: A primer on the next generation of emerging markets. *The Journal of Wealth Management, 13*(3), 50-58.

R Core Team. (2019, 1 6). R: A language and environment for statistical computing. R Foundation for Statistical Computing. Vienna, Austria. Retrieved from URL https://www.R-project.org/

Rambaccussing, D. (2010). Long Memory, Return Predictability and Unconditional Risk: Evidence from African stock Markets. *African Review of Economics and Finance, 1*(2), 72-87.

Ramji, A. Z., Wairimu, V., Mwita, M., & Mwanyasi, R. (2019). East Africa. In H. Raubenheimer, *African Capital Markets Challenges and Opportunities* (pp. 39-57). CFA Institute Research Foundation.

Rao, K. S., & Moseki, K. K. (2011). Analysing volatility in equity indices–A Markov approach for Botswana domestic company indices. *South African Journal of Industrial Engineering, 22*(1), 83-98.

Rydqvist, K., Spizman, J., & Strebulaev, I. (2014). Government policy and ownership of equity securities. *Journal of Financial Economics, 111*(1), 70-85.

Salmon, F. (2012). The formula that killed Wall Street. *Significance, 9*(1), 16-20.

Samarakoon, L. P. (2011). Stock market interdependence, contagion, and the US financial crisis: The case of emerging and frontier markets. *Journal of International Financial Markets, Institutions and Money, 21*(5), 724-742.

Schiereck, D., Freytag, A., Grimm, M., & Bretschneider, W. H. (2018). *Public corporations in Africa: A continental survey on stock exchanges and capital markets performance (No. 2018-013).* Jena Economic Research Papers.

Schipke, A. (2015). Frontier Markets in Asia and Beyond. . In N. Chowdhury, M. Edmonds, & C. Walker, *Frontier and Developing Asia: The Next Generation of Emerging Markets.* Washington, D. C: International Monetary Fund.

Sephton, P. S. (1995). Response surface estimates of the KPSS stationarity test. *Economics Letters, 47*(3-4), 255-261.

Serkin, G. (2015). *Frontier: Exploring the top ten emerging markets of tomorrow.* John Wiley & Sons.

Sharp, D. J. (1991). Uncovering the hidden value in high-risk investments. *MIT Sloan Management Review, 32*(4), 69.

Sherlock, C., Fearnhead, P., & Roberts, G. O. (2010). The random walk Metropolis: linking theory and practice through a case study. *Statistical Science, 25*(2), 172-190.

Shleifer, A., & Vishny, R. (2011). Fire sales in finance and macroeconomics. *Journal of Economic Perspectives, 25*(1), 29-48.

Simpson, J. L. (2010). Were there warning signals from banking sectors for the 2008/2009 global financial crisis? *Applied Financial Economics, 20*(1-2), 45-61.

Song, Y. (2014). Modelling regime switching and structural breaks with an infinite hidden Markov model. *Journal of Applied Econometrics, 29* (5), 825-842.

Sopipan, N., Sattayatham, P., & Premanode, B. (2012). Forecasting volatility of gold price using markov regime switching and trading strategy. *Journal of Mathematical Finance, 2*(01), 121-131.

Speidell, L. S. (2009). Investing in the unknown and the unknowable - Behavioral finance in frontier markets. *The Journal of Behavioral Finance, 10*(1), 1-8.

Speidell, L. S., & Krohne, A. (2007). The case for frontier equity markets. *The Journal of Investing, 16*(3), 12-22.

Spiegelhalter, D., Best, N., Carlin, B. P., & Van Der Linde, A. (2002). Bayesian measures of model complexity and fit. *Journal of the Royal Statistical Society: Series B (Statistical Methodology), 64*(4), 583-639.

Stoyanov, S. V., Rachev, S. T., Racheva-Yotova, B., & Fabozzi, F. J. (2011). Fat-tailed models for risk estimation. *The Journal of Portfolio Management, 37*(2), 107-117.

Suliman, A. H., & Mollick, A. V. (2009). Human capital development, war and foreign direct investment in sub-Saharan Africa. *Oxford Development Studies, 37*(1), 47-61.

Taylor, S. J. (1982a). Financial returns modelled by the product of two stochastic processes — a study of daily sugar prices. In O. Anderson, *Time Series Analysis: Theory and Practice* (Vol. 1, pp. 1961–1979). Amsterdam: North-Holland.

Taylor, S. J. (1994b). Modeling stochastic volatility: A review and comparative study. *Mathematical finance, 4*(2), 183-204.

Teräsvirta, T. (2009). An introduction to univariate GARCH models. In *Handbook of Financial Time Series* (pp. 17-42). Berlin, Heidelberg: Springer.

Than-Thi, H., Dong, M. C., & Chen, C. W. (2019). Bayesian modelling structural changes on Housing Price Dynamics . *International Conference of the Thailand Econometrics Society* (pp. 83-104). Cham: Springer.

Thurner, S., Farmer, J. D., & Geanakoplos, J. (2012). Leverage causes fat tails and clustered volatility. *Quantitative Finance, 12*(5), 695-707.

Tronzano, M. (2001). Macroeconomic Fundamentals and Exchange Rate Credibility. Further Evidence on the Italian Experience from a Regime-switching Approach. *Scottish Journal of Political Economy, 48*(4), 442-460.

Turner, A. L., & Weigel, E. J. (1992). Daily stock market volatility: 1928–1989. *Management Science, 38*(11), 1586-1609.

Uduanu, D. (2019). Nigeria. In H. Raubenheimer, *African Capital Markets Challenges and Opportunities* (pp. 59-71). CFA Institute Research Foundation.

Ushad, S. A., Fowdar, S., Vinesh, S. R., & Jowaheer, M. (2008). Return Distributions: Evidence from Emerging African Stock Exchanges . *ICFAI Journal of Financial Economics, 6*(3), 41-52.

Uyaebo, S. O., Atoi, V. N., & Usman, F. (2015). Nigeria stock market volatility in comparison with some countries: Application of asymmetric GARCH models. *CBN Journal of Applied Statistics, 6*(2), 133-160.

Vandewalle, N., Boveroux, P., Minguet, A., & Ausloos, M. (1998). The crash of October 1987 seen as a phase transition: amplitude and universality . *Physica A: Statistical Mechanics and its Applications, 255*(1-2), 201-210.

Vo, M. T. (2009). Regime-switching stochastic volatility: Evidence from the crude oil market. *Energy Economics, 31*(5), 779-788.

Voth, H. J. (2003). With a bang, not a whimper: Pricking Germany's "stock market bubble" in 1927 and the slide into depression. *The Journal of Economic History, 63* (1), 65-99.

Walid, C., Chaker, A., Masood, O., & Fry, J. (2011). Stock market volatility and exchange rates in emerging countries: A Markov-state switching approach. *Emerging Markets Review, 12*(3), 272-292.

Wang, Y., Wu, C., & Yang, L. (2016). Forecasting crude oil market volatility: A Markov switching multifractal volatility approach. *International Journal of Forecasting, 32*(1), 1-9.

Weron, R., Bierbrauer, M., & Trück, S. (2004). Modeling electricity prices: jump diffusion and regime switching. *Physica A: Statistical Mechanics and its Applications, 336*(1-2), 39-48.

Zaremba, A. (2019). Performance persistence in anomaly returns: Evidence from frontier markets. *Emerging Markets Finance and Trade*, 1-22.

Zaremba, A., & Maydybura, A. (2019). The cross-section of returns in frontier equity markets: Integrated or segmented pricing? *Emerging Markets Review, 38*, 219-238.

Zhang, Q., Sornette, D., Balcilar, M., Gupta, R., Ozdemir, Z. A., & Yetkiner, H. (2016). LPPLS bubble indicators over two centuries of the S&P 500 index. *Physica A: statistical Mechanics and its Applications, 458*, 126-139.

Zivot, E., & Andrews, D. W. (2002). Further evidence on the great crash, the oil-price shock, and the unit-root hypothesis. *Journal of Business & Economic Statistics, 20*(1), 25-44.

APPENDIX A: COMPUTER CODE IN PYTHON

```
import os
os.chdir('C:/Training2018')
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
prices = pd.read_csv('GSEAllShareIndex.csv', index_col = 0, parse_dates =
True)
prices.head()
prices.describe()
prices.plot(subplots = True, layout = (2,2), figsize = (10, 6), sharex = False);
plt.title('Index Levels of the Composite Indices')
prices.plot(subplots = True, layout = (2,2), figsize = (10, 6), sharex = False);
plt.subtitle('Index Levels of the Composite Indices')
plt.suptitle('Index Levels of the Composite Indices')
returnsIndices = np.log(prices).diff().dropna()
returnsIndices.head()
returnsIndices.describe()
np.skew(returnsIndices)
returnsIndices.skew()
returnsIndices.kutosis()
from scipy.stats import kurtosis
kutosis(returnsIndices)
kurtosis(returnsIndices)
fig, ax = plt.subplots(2, 2, figsize = (12, 6), sharex = False, sharey = False)
```



```
ax[0].returnsIndices.GSE.hist(bins = 100, normed = True)

ax[0].returnsIndices['GSE'].hist(bins = 100, normed = True)

returnsIndices.head()

ax[0].returnsIndices.plot(kind = 'hist')

returnsIndices.plot(kind = 'hist')

sns.distplot(returnsIndices['GSE'], kde = False, fit = stats.norm)

import seaborn as sns

sns.distplot(returnsIndices['GSE'], kde = False, fit = stats.norm)

from scipy import stats

sns.distplot(returnsIndices['GSE'], kde = False, fit = stats.norm)

sns.distplot(returnsIndices['GSE'], kde = False, fit = stats.norm, col = 'blue')

sns.distplot(returnsIndices['GSE'], kde = False, fit = stats.norm, color = 'blue')

sns.distplot(returnsIndices['GSE'], kde = False, fit = stats.norm, color = 'blue',

label = 'GSE')

sns.legend()

sns.distplot(returnsIndices['GSE'], kde = False, fit = stats.norm,

rug_kws={"color": "g"}, kde_kws={"color": "k", "lw": 2, "label": "KDE"})

sns.distplot(returnsIndices['GSE'], kde = False, fit = stats.norm,

rug_kws={"color": "g"}, kde_kws={"color": "k", "lw": 2, "label": "KDE"})

sns.distplot(returnsIndices['GSE'], kde = False, fit = stats.norm,

rug_kws={"color": "g"}, kde_kws={"color": "k", "lw": 1, "label": "KDE"})

plt.xlabel('Returns')

plt.ylabel('Frequency')

plt.legend(loc = 'best')

plt.xlabel('Returns of GSE')
```

```
plt.title('GSE Returns Distribution')
sns.distplot(returnsIndices['KSE'], kde = False, fit = stats.norm,
rug_kws={"color": "g"}, kde_kws={"color": "k", "lw": 1, "label": None})
sns.distplot(returnsIndices['KSE'], kde = False, fit = stats.norm,
rug_kws={"color": "g"}, kde_kws={"color": "k", "lw": 1, "label": None})
plt.title('KSE Returns Distribution')
plt.xlabel('Returns of KSE')
plt.ylabel('Frequency')
sns.distplot(returnsIndices['NSE'], kde = False, fit = stats.norm,
rug_kws={"color": "g"}, kde_kws={"color": "k", "lw": 1, "label": None})
sns.distplot(returnsIndices['NSE'], kde = False, fit = stats.norm,
rug_kws={"color": "g"})
plt.ylabel('Frequency')
plt.xlabel('Returns of NSE')
plt.title('NSE Returns Distribution')
sns.distplot(returnsIndices['BSI'], kde = False, fit = stats.norm,
rug_kws={"color": "g"})
sns.distplot(returnsIndices['BSI'], kde = False, fit = stats.norm,
rug_kws={"color": "g"})
plt.title('BSI Returns Distribution')
plt.xlabel('Returns of BSI')
plt.ylabel('Frequency')
sns.distplot(returnsIndices, kde = False, fit = stats.norm, rug_kws={"color":
"g"})
```



```
returnsIndices.plot(subplots = True, layout = (2,2), figsize = (10, 6), sharex =
False);
returnsIndices.plot(subplots = True, kind = 'hist', layout = (2,2), figsize = (10,
6), sharex = False);
from statsmodels.tsa.stattools import adfuller
adfuller(returnsIndices['GSE'])
adfuller(returnsIndices['KSE'])
adfuller(returnsIndices['NSE'])
adfuller(returnsIndices['BSI'])
from statsmodels.tsa.stattools import kpss
kpss(returnsIndices['GSE'])
kpss(returnsIndices['KSE'])
kpss(returnsIndices['NSE'])
kpss(returnsIndices['BSI'])
from arch.unitroot import PhillipsPerron
get_ipython().run_line_magic('pip', 'install arch')
from arch.unitroot import PhillipsPerron
PhillipsPerron(returnsIndices['GSE'])
PhillipsPerron(returnsIndices['KSE'])
PhillipsPerron(returnsIndices['GSE'])
PhillipsPerron(returnsIndices['NSE'])
PhillipsPerron(returnsIndices['BSI'])
from arch.unitroot import kpss
from arch.unitroot import KPSS
KPSS(returnsIndices['BSI'])
```

```
from arch.unitroot import ZivotAndrews
ZivotAndrews(returnsIndices['GSE'])
ZivotAndrews(returnsIndices['KSE'])
ZivotAndrews(returnsIndices['NSE'])
ZivotAndrews(returnsIndices['BSI'])
import ruptures as rpt
pointsGSE = np.array(returnsIndicies['GSE'])
pointsGSE = np.array(returnsIndices['GSE'])
model = 'l2'
algo = rpt.Binseg(model=model).fit(pointsGSE)
bkpsGSE = algo.predict(n_bkps=10)
rpt.show.display(points, my_bkpsGSE, figsize=(10, 6))
rpt.show.display(pointsGSE, my_bkpsGSE, figsize=(10, 6))
rpt.show.display(pointsGSE, bkpsGSE, figsize=(10, 6))
pointsGSE = np.array(prices['GSE'])
algo = rpt.Binseg(model=model).fit(pointsGSE)
bkpsGSE = algo.predict(n_bkps=10)
rpt.show.display(pointsGSE, bkpsGSE, figsize=(10, 6))
plt.title('Change Point Detection: GSE Composite Index')
rpt.show.display(pointsGSE, bkpsGSE, figsize=(8, 6))
rpt.show.display(pointsGSE, bkpsGSE, figsize=(8, 4))
plt.title('Change Point Detection: GSE Composite Index')
plt.xlabel('Time Index')
plt.ylabel('Composite Level')
fig, ax = plt.subplots(2,2, figsize = (12, 6), sharey = False)
```



```
ax[0].rpt.show.display(pointsGSE, bkpsGSE, figsize=(8, 4))
returnsIndices.plot.hist(subplots=True, layout=(2,2), figsize=(10, 10),
bins=100)
returnsIndices.plot.hist(subplots=True, layout=(2,2), figsize=(10, 10),
bins=200)
returnsIndices.plot.hist(subplots=True, layout=(2,2), figsize=(10, 10), normed
= True, bins=200)
returnsIndices.plot.hist(subplots=True, layout=(2,2), figsize=(10, 10), density
= True, bins=200)
returnsIndices.plot.hist(subplots=True, layout=(2,2), figsize=(10, 10), density
= False, bins=200)
returnsIndices.plot.hist(subplots=True, layout=(2,2), figsize=(10, 10), density
= True, bins=200)
returnsIndices.plot.hist(subplots=True, layout=(2,2), figsize=(10, 10), density
= True, bins=200, sharexy = False)
returnsIndices.plot.hist(subplots=True, layout=(2,2), figsize=(10, 10), density
= True, bins=200, sharey = False)
returnsIndices.plot.hist(subplots=True, layout=(2,2), figsize=(10, 10), density
= True, bins=200, sharey = False, sharex = False)
demGSE = returnsIndices['GSE'] - returnsIndices['GSE'].mean()
demGSE.head()
demKSE = returnsIndices['KSE'] - returnsIndices['KSE'].mean()
demNSE = returnsIndices['NSE'] - returnsIndices['NSE'].mean()
demBSI = returnsIndices['BSI'] - returnsIndices['BSI'].mean()
from statsmodels.stats.diagnostic import het_arch
```

```
het_arch(demGSE)
het_arch(demKSE)
het_arch(demNSE)
het_arch(demBSI)
from statsmodels.stats.diagnostic import acorr_ljungbox
acorr_ljungbox(demBSI)
acorr_ljungbox(demBSI, lags = [10], return_df = True)
get_ipython().run_line_magic('pip', 'install pypr')
from pypr.stattest.ljungbox import lbqtest
from pypr.stattest.ljungbox import *
acorr_ljungbox(demBSI, lags = [10], return_df = True)
acorr_ljungbox(demBSI)
from statsmodels.stats.diagnostic import acorr_breusch_godfrey
acorr_breusch_godfrey(demNSE)
demNSE = returnsIndices['NSE'] - returnsIndices['NSE'].mean()
het_arch(demNSE)
het_arch(returnsIndices['NSE'])
acorr_ljungbox(demBSI, lags = [10], boxpierce = True)
acorr_ljungbox(demBSI, lags = None, boxpierce = True)
acorr_ljungbox(demBSI, lags = [10], boxpierce = True)
acorr_ljungbox(demNSE, lags = [10], boxpierce = True)
acorr_ljungbox(demKSE, lags = [10], boxpierce = True)
acorr_ljungbox(demGSE, lags = [10], boxpierce = True)
het_arch(returnsIndices['NSE'])
het_arch(demBSI)
```


het_arch(demGSE)

--- Heavy-tails-------------------------------------

```
import os
os.chdir('C:/Training2018')
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
prices = pd.read_csv('GSEAllShareIndex.csv', index_col = 0, parse_dates =
True)
get_ipython().run_line_magic('pylab', '')
import quantecon as qe
returns = np.log().diff(prices).dropna()
returns = np.log(prices).diff().dropna()
returns.head()
gseReturns = returns.GSE
kseReturns = returns.KSE
nseReturns = returns.NSE
bsiReturns = returns.BSI
gseReturns.head()
fig, ax = plt.subplots()
ax.plot(gseReturns, linestyle = '', marker = 'o', alpha = 0.8, ms = 4)
ax.vlines(gseReturns.index, 0, gseReturns.values, lw = 0.3)
ax.set_ylabel('Returns', fontsize = 12)
ax.set_xlabel('Date', fontsize = 12)
xl, xr = ax.get_xlim()
```

```
gseStd = gseReturns.std()
ax.hlines([-3*gseStd,0,3*gseStd], xl, xr, linestyle = '--', lw = 2, color = 'r')
fig, ax = plt.subplots()
ax.plot(kseReturns, linestyle = '', marker = 'o', alpha = 0.8, ms = 4)
ax.vlines(kseReturns.index, 0, kseReturns.values, lw = 0.3)
ax.set_xlabel('Date', fontsize = 12)
ax.set_ylabel('Returns', fontsize = 12)
kseStd = kseReturns.std()
xl, xr = ax.get_xlim()
ax.hlines([-3*kseStd,0,3*kseStd], xl, xr, linestyle = '--', lw = 2, color = 'r')
fig, ax = plt.subplots()
ax.plot(nseReturns, linestyle = '', marker = 'o', alpha = 0.8, ms = 4)
ax.vlines(nseReturns.index, 0, nseReturns.values, lw = 0.3)
xl, xr = ax.get_xlim()
nseStd = nseReturns.std()
ax.hlines([-3*nseStd, 0, 3*nseStd], xl, xr, linestyle = '--', lw = 2, color = 'r')
fig, ax = plt.subplots()
ax.plot(bsiReturns, linestyle = '', marker = 'o', alpha = 0.8, ms = 4)
bsiStd = bsiReturns.std()
xl, xr = ax.get_xlim()
ax.hlines([-3*bsiStd, 0, 3*bsiStd], xl, xr, linestyle = '--', lw = 2, color = 'r')
ax.set_ylabel('Returns', fontsize = 12)
ax.set_xlabel('Date', fontsize = 12)
fig, ax = plt.subplots()
ax.vlines(nseReturns.index, 0, nseReturns.values, lw = 0.3)
```

```
fig, ax = plt.subplots()
ax.plot(nseReturns, linestyle = '', marker = 'o', alpha = 0.8, ms = 4)
ax.vlines(nseReturns.index, 0, nseReturns.values, lw = 0.3)
ax.set_xlabel('Date', fontsize = 12)
ax.set_ylabel('Returns', fontsize = 12)
xl, xr = ax.get_xlim()
ax.hlines([-3*nseStd, 0, 3*nseStd], xl, xr, linestyle = '--', lw = 2, color = 'r')
```

-- QQ ------------------------

```
# coding: utf-8
import numpy as np
import pandas as pd
import scipy.stats as scs
import matplotlib.pyplot as plt
plt.style.use('bmh')
get_ipython().run_line_magic('pylab', '')
import os
os.chdir('C:/Training2018')
prices = pd.read_csv('GSEAllShareIndex.csv', index_col = 0, parse_dates =
True)
returns = np.log(prices).diff().dropna()
scs.probplot(returns.GSE, dist=scipy.stats.norm,
plot=plt.figure().add_subplot(111))
scs.probplot(returns.GSE, dist = scs.norm, plot=plt.figure().add_subplot(111))
plt.title('')
plt.grid(b = None)
```



```
scs.probplot(returns.KSE, dist = scs.norm, plot=plt.figure().add_subplot(111));
plt.title('')
plt.grid(b = None)
scs.probplot(returns.NSE, dist = scs.norm, plot=plt.figure().add_subplot(111));
plt.title('')
plt.grid(b = None)
scs.probplot(returns.BSI, dist = scs.norm, plot=plt.figure().add_subplot(111));
plt.title('')
plt.grid(b = None)
returns.to_csv('GSEAllShareReturns.csv')
get_ipython().run_line_magic('save', 'PhDAdditional 1-25')
------------------------ Value at Risk – Hill estimator
import numpy as np
import pandas as pd
import scipy.stats as scs
import os
data = pr.read_csv('KSEIndex.csv', index_col = 0, parse_dates = True)
data = pd.read_csv('KSEIndex.csv', index_col = 0, parse_dates = True)
data.head()
data['ret'] = np.log(data['KSE']).diff()
data.head()
data.drop(inplace = True)
data.dropna(inplace = True)
data.head()
mean = np.mean(data['ret'])
```

```
std_dev = np.std(data['ret'])

VaR_90 = norm.ppf(0.1, mean, std_dev)

VaR_90 = scs.norm.ppf(0.1, mean, std_dev)

VaR_95 = scs.norm.ppf(0.05, mean, std_dev)

VaR_99 = scs.norm.ppf(0.01, mean, std_dev)

VaR_90

VaR_95

VaR_99

VaR_90h = data['ret'].quantile(0.1)

VaR_95h = data['ret'].quantile(0.05)

VaR_99h = data['ret'].quantile(0.01)

VaR_90h

VaR_95h

VaR_99h

threshold = -0.025

alpha = 0.01

data1 = data[data['ret'] <= threshold]

data1 = data1.sort_values(['ret'])

data['k'] = data1.iloc[0]['ret']

xi = (np.log(-data['k']) - np.log(-data1['ret'])).sum()*(1/(data1.shape[0] - 1))

xi

------------------------------------- Smooth plots-----------------------------------

import numpy as np

import pandas as pd

import statsmodels.api as sm
```

```
import matplotlib.pyplot as plt

import os

os.chdir('C:/Training2018')

prices = pd.read_csv('GSEAllShareIndex.csv', index_col = 0, parse_dates =
True)

returns = np.log(prices).diff().dropna()

retGSE = returns['GSE']

retGSE = retGSE - mean(retGSE)

retGSE.head()

modGSE = sm.tsa.MarkovRegression(retGSE, k_regimes = 2, trend = 'nc',
switching_variance = True).fit()

modGSE.summary()

fig, axes = plt.subplots(2, figsize = (12, 8))

ax = axes[0]

ax.plot(modGSE.smooth_marginal_probabilities[0])

ax.plot(modGSE.smoothed_marginal_probabilities[0])

monthGSE = retGSE.resample('M')

modGSE = sm.tsa.MarkovRegression(monthGSE, k_regimes = 2, trend = 'nc',
switching_variance = True).fit()

monthGSE.head()

monthGSE

monthGSE.mean()

retGSEm = monthGSE.mean()

fig, axes = plt.subplots(2, figsize = (12, 8))
```

```
modGSE = sm.tsa.MarkovRegression(retGSEm, k_regimes = 2, trend = 'nc',
switching_variance = True).fit()
ax = axes[0]
ax.plot(modGSE.smoothed_marginal_probabilities[0])
ax.set(title='Smoothed probability of a low-variance regime for GSE returns')
ax = axes[1]
ax.plot(modGSE.smoothed_marginal_probabilities[1])
ax.set(title='Smoothed probability of a high-variance regime for GSE returns')
retKSE = returns['KSE']
retKSE = retKSE - mean(retKSE)
retKSEm = retKSE.resample('M').mean()
modKSE = sm.tsa.MarkovRegression(retKSEm, k_regimes = 2, trend = 'nc',
switching_variance = True).fit()
fig, axes = plt.subplots(2, figsize = (12, 8))
ax = axes[0]
ax.plot(modKSE.smoothed_marginal_probabilities[0])
ax.set(title='Smoothed probability of a low-variance regime for stock returns')
fig, axes = plt.subplots(2, figsize = (12, 8))
ax = axes[0]
ax.plot(modKSE.smoothed_marginal_probabilities[0])
ax.set(title='Smoothed probability of a low-variance regime for KSE returns')
ax = axes[1]
ax.plot(modKSE.smoothed_marginal_probabilities[1])
ax.set(title='Smoothed probability of a high-variance regime for KSE returns')
returns.head()
```

```
retGSE = returns['GSE']
retGSE = retGSE - mean(retGSE)
retGSE = retGSE.resample('W').median()
retGSE.head()
modGSE = sm.tsa.MarkovRegression(retGSE, k_regimes = 2, trend='nc',
switching_variance=True).fit()
fig, axes = plt.subplots(2, figsize=(12,8))
ax = axes[0]
ax.plot(modGSE.smoothed_marginal_probabilities[0])
ax.set(title='Smoothed probability of a low-variance regime for GSE Index
returns')
ax = axes[1]
ax.plot(modGSE.smoothed_marginal_probabilities[1])
ax.set(title='Smoothed probability of a high-variance regime for GSE Index
returns')
retKSE = returns['KSE']
retKSE = retKSE - mean(retKSE)
retKSE = retKSE.resample('W').median()
retKSE.head()
modKSE = sm.tsa.MarkovRegression(retKSE, k_regimes = 2, trend='nc',
switching_variance=True).fit()
fig, axes = plt.subplots(2, figsize=(12,8))
ax = axes[0]
ax.plot(modKSE.smoothed_marginal_probabilities[0])
```

```
ax.set(title='Smoothed probability of a low-variance regime for KSE Index
returns')
ax = axes[1]
ax.plot(modKSE.smoothed_marginal_probabilities[1])
ax.set(title='Smoothed probability of a high-variance regime for KSE Index
returns')
retNSE = returns['NSE']
retNSE = retNSE - mean(retNSE)
retNSE = retNSE.resample('W').mean()
retNSE.head()
modNSE = sm.tsa.MarkovRegression(retNSE, k_regimes = 2, trend='nc',
switching_variance=True).fit()
fig, axes = plt.subplots(2, figsize=(12,8))
ax = axes[0]
ax.plot(modNSE.smoothed_marginal_probabilities[0])
ax.set(title='Smoothed probability of a low-variance regime for NSE Index
returns')
ax = axes[1]
ax.plot(modNSE.smoothed_marginal_probabilities[1])
ax.set(title='Smoothed probability of a high-variance regime for NSE Index
returns')
retBSI = returns['BSI']
retBSI = retBSI - mean(retBSI)
retBSI = retBSI.resample('W').mean()
retBSI.head()
```

```
modBSI = sm.tsa.MarkovRegression(retBSI, k_regimes = 2, trend='nc',
switching_variance=True).fit()
fig, axes = plt.subplots(2, figsize=(12,8))
ax = axes[0]
ax.plot(modBSI.smoothed_marginal_probabilities[0])
ax.set(title='Smoothed probability of a low-variance regime for BSI Index
returns')
ax = axes[1]
ax.plot(modBSI.smoothed_marginal_probabilities[1])
ax.set(title='Smoothed probability of a high-variance regime for BSI Index
returns')
get_ipython().run_line_magic('save', 'PhDSmoothPlots 1-116')
```

APPENDIX B: COMPUTER CODE IN R

```
setwd('C:/Training2018')

getwd()

levels = read.csv('GSEAllShareIndex.csv', header = T)

head(levels, 10)

class(levels)

date <- levels$Date

date <- as.Date(date, '%m/%d/%Y')

head(date)

index = levels[, c(2, 3, 4, 5)]

head(index)

library(xts)

data = xts(index, date)

head(data)

returnsIndices = (diff(log(data)))[-1, ]

head(returnsIndices)

tail(returnsIndices)

head(returnsIndices[,'GSE'])


#############################Start

models##################################


# Single models for GSE

retGSE = (returnsIndices[,'GSE'] - mean(returnsIndices[,'GSE'])) * 100

head(retGSE)
```

```
# Single regime models
#GARCH(1, 1) with normal distributions
specSGSEnorm <- CreateSpec(variance.spec = list(model =
c('sGARCH')),distribution.spec = list(distribution = c('norm')),switch.spec =
list(do.mix = FALSE))
fitSGSEnorm = FitMCMC(specSGSEnorm, data = retGSE, ctr = list())
summary(fitSGSEnorm)
#GARCH(1, 1) with skew normal distributions
specSGSEsnorm <- CreateSpec(variance.spec = list(model =
c('sGARCH')),distribution.spec = list(distribution = c('snorm')),switch.spec =
list(do.mix = FALSE))
fitSGSEsnorm = FitMCMC(specSGSEsnorm, data = retGSE, ctr = list())
summary(fitSGSEsnorm)
#GARCH(1, 1) with std distributions
specSGSEstd <- CreateSpec(variance.spec = list(model =
c('sGARCH')),distribution.spec = list(distribution = c('std')),switch.spec =
list(do.mix = FALSE))
fitSGSEstd = FitMCMC(specSGSEstd, data = retGSE, ctr = list())
summary(fitSGSEstd)
#GARCH(1, 1) with sstd distributions
specSGSEsstd <- CreateSpec(variance.spec = list(model =
c('sGARCH')),distribution.spec = list(distribution = c('sstd')),switch.spec =
list(do.mix = FALSE))
fitSGSEsstd = FitMCMC(specSGSEsstd, data = retGSE, ctr = list())
summary(fitSGSEsstd)
```

```
#GARCH(1, 1) with ged distributions
specSGSEged <- CreateSpec(variance.spec = list(model =
c('sGARCH')),distribution.spec = list(distribution = c('ged')),switch.spec =
list(do.mix = FALSE))
fitSGSEged = FitMCMC(specSGSEged, data = retGSE, ctr = list())
summary(fitSGSEged)
#GARCH(1, 1) with skewed ged distributions
spec1SGSEsged <- CreateSpec(variance.spec = list(model =
c('sGARCH')),distribution.spec = list(distribution = c('sged')),switch.spec =
list(do.mix = FALSE))
fitSGSEsged = FitMCMC(spec1SGSEsged, data = retGSE, ctr = list())
summary(fitSGSEsged)
#GARCH(1, 1) with normal distributions KSE
retKSE = (returnsIndices[,'KSE'] - mean(returnsIndices[,'KSE'])) * 100
head(retKSE)
specSKSEnorm <- CreateSpec(variance.spec = list(model =
c('sGARCH')),distribution.spec = list(distribution = c('norm')),switch.spec =
list(do.mix = FALSE))
fitSKSEnorm = FitMCMC(specSKSEnorm, data = retKSE, ctr = list())
summary(fitSKSEnorm)
#GARCH(1, 1) with skew normal distributions
```

```
specSKSEsnorm <- CreateSpec(variance.spec = list(model =
c('sGARCH')),distribution.spec = list(distribution = c('snorm')),switch.spec =
list(do.mix = FALSE))
fitSKSEsnorm = FitMCMC(specSKSEsnorm, data = retKSE, ctr = list())
summary(fitSKSEsnorm)
#GARCH(1, 1) with std distributions
specSKSEstd <- CreateSpec(variance.spec = list(model =
c('sGARCH')),distribution.spec = list(distribution = c('std')),switch.spec =
list(do.mix = FALSE))
fitSKSEstd = FitMCMC(specSKSEstd, data = retKSE, ctr = list())
summary(fitSKSEstd)
#GARCH(1, 1) with sstd distributions
spec1SKSEsstd <- CreateSpec(variance.spec = list(model =
c('sGARCH')),distribution.spec = list(distribution = c('sstd')),switch.spec =
list(do.mix = FALSE))
fit1SKSEsstd = FitMCMC(spec1SKSEsstd, data = retKSE, ctr = list())
summary(fit1SKSEsstd)
#GARCH(1, 1) with ged distributions
specSKSEged <- CreateSpec(variance.spec = list(model =
c('sGARCH')),distribution.spec = list(distribution = c('ged')),switch.spec =
list(do.mix = FALSE))
fitSKSEged = FitMCMC(specSKSEged, data = retKSE, ctr = list())
summary(fitSKSEged)
#GARCH(1, 1) with sged distributions
```

```
specSKSEsged <- CreateSpec(variance.spec = list(model =
c('sGARCH')),distribution.spec = list(distribution = c('sged')),switch.spec =
list(do.mix = FALSE))
fitSKSEsged = FitMCMC(specSKSEsged, data = retKSE, ctr = list())
summary(fitSKSEsged)
######################
#GARCH(1, 1) with normal distributions NSE
retNSE = (returnsIndices[,'NSE'] - mean(returnsIndices[,'NSE'])) * 100
head(retNSE)
specSNSEnorm <- CreateSpec(variance.spec = list(model =
c('sGARCH')),distribution.spec = list(distribution = c('norm')),switch.spec =
list(do.mix = FALSE))
fitSNSEnorm = FitMCMC(specSNSEnorm, data = retNSE, ctr = list())
summary(fitSNSEnorm)
#GARCH(1, 1) with skew normal distributions
specSNSEsnorm <- CreateSpec(variance.spec = list(model =
c('sGARCH')),distribution.spec = list(distribution = c('snorm')),switch.spec =
list(do.mix = FALSE))
fitSNSEsnorm = FitMCMC(specSNSEsnorm, data = retNSE, ctr = list())
summary(fitSNSEsnorm)
#GARCH(1, 1) with std distributions
specSNSEstd <- CreateSpec(variance.spec = list(model =
c('sGARCH')),distribution.spec = list(distribution = c('std')),switch.spec =
list(do.mix = FALSE))
fitSNSEstd = FitMCMC(specSNSEstd, data = retNSE, ctr = list())
```



```
summary(fitSNSEstd)

#GARCH(1, 1) with sstd distributions

specSNSEsstd <- CreateSpec(variance.spec = list(model =
c('sGARCH')),distribution.spec = list(distribution = c('sstd')),switch.spec =
list(do.mix = FALSE))

fitSNSEsstd = FitMCMC(specSNSEsstd, data = retNSE, ctr = list())

summary(fitSNSEsstd)

#GARCH(1, 1) with ged distributions

specSNSEged <- CreateSpec(variance.spec = list(model =
c('sGARCH')),distribution.spec = list(distribution = c('ged')),switch.spec =
list(do.mix = FALSE))

fitSNSEged = FitMCMC(specSNSEged, data = retNSE, ctr = list())

summary(fitSNSEged)

#GARCH(1, 1) with sged distributions

specSNSEsged <- CreateSpec(variance.spec = list(model =
c('sGARCH')),distribution.spec = list(distribution = c('sged')),switch.spec =
list(do.mix = FALSE))

fitSNSEsged = FitMCMC(specSNSEsged, data = retNSE, ctr = list())

summary(fitSNSEsged)

##############################################

#GARCH(1, 1) with normal distributions BSI

retBSI = (returnsIndices[,'BSI'] - mean(returnsIndices[,'BSI'])) * 100

head(retBSI)
```

```
specSBSInorm <- CreateSpec(variance.spec = list(model =
c('sGARCH')),distribution.spec = list(distribution = c('norm')),switch.spec =
list(do.mix = FALSE))
fitSBSInorm = FitMCMC(specSBSInorm, data = retBSI, ctr = list())
summary(fitSBSInorm)
#GARCH(1, 1) with skew normal distributions
specSBSIsnorm <- CreateSpec(variance.spec = list(model =
c('sGARCH')),distribution.spec = list(distribution = c('snorm')),switch.spec =
list(do.mix = FALSE))
fitSBSIsnorm = FitMCMC(specSBSIsnorm, data = retBSI, ctr = list())
summary(fitSBSIsnorm)
#GARCH(1, 1) with std distributions
specSBSIstd <- CreateSpec(variance.spec = list(model =
c('sGARCH')),distribution.spec = list(distribution = c('std')),switch.spec =
list(do.mix = FALSE))
fitSBSIstd = FitMCMC(specSBSIstd, data = retBSI, ctr = list())
summary(fitSBSIstd)
#GARCH(1, 1) with sstd distributions
specSBSIsstd <- CreateSpec(variance.spec = list(model =
c('sGARCH')),distribution.spec = list(distribution = c('sstd')),switch.spec =
list(do.mix = FALSE))
fitSBSIsstd = FitMCMC(specSBSIsstd, data = retBSI, ctr = list())
summary(fitSBSIsstd)
#GARCH(1, 1) with ged distributions
```

```
specSBSIged <- CreateSpec(variance.spec = list(model =
c('sGARCH')),distribution.spec = list(distribution = c('ged')),switch.spec =
list(do.mix = FALSE))
fitSBSIged = FitMCMC(specSBSIged, data = retBSI, ctr = list())
summary(fitSBSIged)
#GARCH(1, 1) with skewed ged distributions
spec1SBSIsged <- CreateSpec(variance.spec = list(model =
c('sGARCH')),distribution.spec = list(distribution = c('sged')),switch.spec =
list(do.mix = FALSE))
fit1SBSIsged = FitMCMC(spec1SBSIsged, data = retBSI, ctr = list())
summary(fit1SBSIsged)
###########################################
# Single regime models
#eGARCH(1, 1) with normal distributions
speceGSEnorm <- CreateSpec(variance.spec = list(model =
c('eGARCH')),distribution.spec = list(distribution = c('norm')),switch.spec =
list(do.mix = FALSE))
fiteGSEnorm = FitMCMC(speceGSEnorm, data = retGSE, ctr = list())
summary(fiteGSEnorm)
#GARCH(1, 1) with skew normal distributions
speceGSEsnorm <- CreateSpec(variance.spec = list(model =
c('eGARCH')),distribution.spec = list(distribution = c('snorm')),switch.spec =
list(do.mix = FALSE))
fiteGSEsnorm = FitMCMC(speceGSEsnorm, data = retGSE, ctr = list())
summary(fiteGSEsnorm)
```

```
#GARCH(1, 1) with std distributions
speceGSEstd <- CreateSpec(variance.spec = list(model =
c('eGARCH')),distribution.spec = list(distribution = c('std')),switch.spec =
list(do.mix = FALSE))
fiteGSEstd = FitMCMC(speceGSEstd, data = retGSE, ctr = list())
summary(fiteGSEstd)
#GARCH(1, 1) with sstd distributions
speceGSEsstd <- CreateSpec(variance.spec = list(model =
c('eGARCH')),distribution.spec = list(distribution = c('sstd')),switch.spec =
list(do.mix = FALSE))
fiteGSEsstd = FitMCMC(speceGSEsstd, data = retGSE, ctr = list())
summary(fiteGSEsstd)
#GARCH(1, 1) with ged distributions
speceGSEged <- CreateSpec(variance.spec = list(model =
c('eGARCH')),distribution.spec = list(distribution = c('ged')),switch.spec =
list(do.mix = FALSE))
fiteGSEged = FitMCMC(speceGSEged, data = retGSE, ctr = list())
summary(fiteGSEged)
#GARCH(1, 1) with sged distributions
speceGSE1sged <- CreateSpec(variance.spec = list(model =
c('sGARCH')),distribution.spec = list(distribution = c('sged')),switch.spec =
list(do.mix = FALSE))
fiteGSE1sged = FitMCMC(speceGSEsged, data = retGSE, ctr = list())
summary(fiteGSE1sged)
```

```
#########################################
#eGARCH(1, 1) with normal distributions KSE
speceKSEnorm <- CreateSpec(variance.spec = list(model =
c('eGARCH')),distribution.spec = list(distribution = c('norm')),switch.spec =
list(do.mix = FALSE))
fiteKSEnorm = FitMCMC(speceKSEnorm, data = retKSE, ctr = list())
summary(fiteKSEnorm)
#GARCH(1, 1) with skew normal distributions
speceKSEsnorm <- CreateSpec(variance.spec = list(model =
c('eGARCH')),distribution.spec = list(distribution = c('snorm')),switch.spec =
list(do.mix = FALSE))
fiteKSEsnorm = FitMCMC(speceKSEsnorm, data = retKSE, ctr = list())
summary(fiteKSEsnorm)
#GARCH(1, 1) with std distributions
speceKSEstd <- CreateSpec(variance.spec = list(model =
c('eGARCH')),distribution.spec = list(distribution = c('std')),switch.spec =
list(do.mix = FALSE))
fiteKSEstd = FitMCMC(speceKSEstd, data = retKSE, ctr = list())
summary(fiteKSEstd)
#GARCH(1, 1) with sstd distributions
speceKSEsstd <- CreateSpec(variance.spec = list(model =
c('eGARCH')),distribution.spec = list(distribution = c('sstd')),switch.spec =
list(do.mix = FALSE))
fiteKSEsstd = FitMCMC(speceKSEsstd, data = retKSE, ctr = list())
summary(fiteKSEsstd)
```

```
#GARCH(1, 1) with ged distributions
speceKSEged <- CreateSpec(variance.spec = list(model =
c('eGARCH')),distribution.spec = list(distribution = c('ged')),switch.spec =
list(do.mix = FALSE))
fiteKSEged = FitMCMC(speceKSEged, data = retKSE, ctr = list())
summary(fiteKSEged)
#GARCH(1, 1) with sged distributions
speceKSEsged <- CreateSpec(variance.spec = list(model =
c('eGARCH')),distribution.spec = list(distribution = c('sged')),switch.spec =
list(do.mix = FALSE))
fiteKSEsged = FitMCMC(speceKSEsged, data = retKSE, ctr = list())
summary(fiteKSEsged)
######################
#eGARCH(1, 1) with normal distributions NSE
speceNSEnorm <- CreateSpec(variance.spec = list(model =
c('eGARCH')),distribution.spec = list(distribution = c('norm')),switch.spec =
list(do.mix = FALSE))
fiteNSEnorm = FitMCMC(speceNSEnorm, data = retNSE, ctr = list())
summary(fiteNSEnorm)
#eGARCH(1, 1) with skew normal distributions
speceNSEsnorm <- CreateSpec(variance.spec = list(model =
c('eGARCH')),distribution.spec = list(distribution = c('snorm')),switch.spec =
list(do.mix = FALSE))
fiteNSEsnorm = FitMCMC(speceNSEsnorm, data = retNSE, ctr = list())
summary(fiteNSEsnorm)
```

```
#eGARCH(1, 1) with std distributions
speceNSEstd <- CreateSpec(variance.spec = list(model =
c('eGARCH')),distribution.spec = list(distribution = c('std')),switch.spec =
list(do.mix = FALSE))
fiteNSEstd = FitMCMC(speceNSEstd, data = retNSE, ctr = list())
summary(fiteNSEstd)
#GARCH(1, 1) with sstd distributions
speceNSEsstd <- CreateSpec(variance.spec = list(model =
c('eGARCH')),distribution.spec = list(distribution = c('sstd')),switch.spec =
list(do.mix = FALSE))
fiteNSEsstd = FitMCMC(speceNSEsstd, data = retNSE, ctr = list())
summary(fiteNSEsstd)
#eGARCH(1, 1) with ged distributions
speceNSEged <- CreateSpec(variance.spec = list(model =
c('eGARCH')),distribution.spec = list(distribution = c('ged')),switch.spec =
list(do.mix = FALSE))
fiteNSEged = FitMCMC(speceNSEged, data = retNSE, ctr = list())
summary(fiteNSEged)
#GARCH(1, 1) with sged distributions
speceNSEsged <- CreateSpec(variance.spec = list(model =
c('eGARCH')),distribution.spec = list(distribution = c('sged')),switch.spec =
list(do.mix = FALSE))
fiteNSEsged = FitMCMC(speceNSEsged, data = retNSE, ctr = list())
summary(fiteNSEsged)
```

```
###############################################
#eGARCH(1, 1) with normal distributions BSI
speceBSInorm <- CreateSpec(variance.spec = list(model =
c('eGARCH')),distribution.spec = list(distribution = c('norm')),switch.spec =
list(do.mix = FALSE))
fiteBSInorm = FitMCMC(speceBSInorm, data = retBSI, ctr = list())
summary(fiteBSInorm)
#eGARCH(1, 1) with skew normal distributions
speceBSIsnorm <- CreateSpec(variance.spec = list(model =
c('eGARCH')),distribution.spec = list(distribution = c('snorm')),switch.spec =
list(do.mix = FALSE))
fiteBSIsnorm = FitMCMC(speceBSIsnorm, data = retBSI, ctr = list())
summary(fiteBSIsnorm)
#eGARCH(1, 1) with std distributions
speceBSIstd <- CreateSpec(variance.spec = list(model =
c('eGARCH')),distribution.spec = list(distribution = c('std')),switch.spec =
list(do.mix = FALSE))
fiteBSIstd = FitMCMC(speceBSIstd, data = retBSI, ctr = list())
summary(fiteBSIstd)
#eGARCH(1, 1) with sstd distributions
speceBSIsstd <- CreateSpec(variance.spec = list(model =
c('eGARCH')),distribution.spec = list(distribution = c('sstd')),switch.spec =
list(do.mix = FALSE))
fiteBSIsstd = FitMCMC(speceBSIsstd, data = retBSI, ctr = list())
summary(fiteBSIsstd)
```

```
#eGARCH(1, 1) with ged distributions
speceBSIged <- CreateSpec(variance.spec = list(model =
c('eGARCH')),distribution.spec = list(distribution = c('ged')),switch.spec =
list(do.mix = FALSE))
fiteBSIged = FitMCMC(speceBSIged, data = retBSI, ctr = list())
summary(fiteBSIged)
#eGARCH(1, 1) with sged distributions
speceBSIsged <- CreateSpec(variance.spec = list(model =
c('eGARCH')),distribution.spec = list(distribution = c('sged')),switch.spec =
list(do.mix = FALSE))
fiteBSIsged = FitMCMC(speceBSIsged, data = retBSI, ctr = list())
summary(fiteBSIsged)
#################################################
# Single regime models
#gjrGARCH(1, 1) with normal distributions
specgjrGSEnorm <- CreateSpec(variance.spec = list(model =
c('gjrGARCH')),distribution.spec = list(distribution = c('norm')),switch.spec =
list(do.mix = FALSE))
fitgjrGSEnorm = FitMCMC(specgjrGSEnorm, data = retGSE, ctr = list())
summary(fitgjrGSEnorm)
#GARCH(1, 1) with skew normal distributions
specgjrGSEsnorm <- CreateSpec(variance.spec = list(model =
c('gjrGARCH')),distribution.spec = list(distribution = c('snorm')),switch.spec =
list(do.mix = FALSE))
fitgjrGSEsnorm = FitMCMC(specgjrGSEsnorm, data = retGSE, ctr = list())
```



```
summary(fitgjrGSEsnorm)

#GARCH(1, 1) with std distributions

specgjrGSEstd <- CreateSpec(variance.spec = list(model =
c('gjrGARCH')),distribution.spec = list(distribution = c('std')),switch.spec =
list(do.mix = FALSE))

fitgjrGSEstd = FitMCMC(specgjrGSEstd, data = retGSE, ctr = list())

summary(fitgjrGSEstd)

#gjrGARCH(1, 1) with sstd distributions

specgjrGSEsstd <- CreateSpec(variance.spec = list(model =
c('gjrGARCH')),distribution.spec = list(distribution = c('sstd')),switch.spec =
list(do.mix = FALSE))

fitgjrGSEsstd = FitMCMC(specgjrGSEsstd, data = retGSE, ctr = list())

summary(fitgjrGSEsstd)

#gjrGARCH(1, 1) with ged distributions

specgjrGSEged <- CreateSpec(variance.spec = list(model =
c('gjrGARCH')),distribution.spec = list(distribution = c('ged')),switch.spec =
list(do.mix = FALSE))

fitgjrGSEged = FitMCMC(specgjrGSEged, data = retGSE, ctr = list())

summary(fitgjrGSEged)

#gjrGARCH(1, 1) with sged distributions

specgjrGSEsged <- CreateSpec(variance.spec = list(model =
c('gjrGARCH')),distribution.spec = list(distribution = c('sged')),switch.spec =
list(do.mix = FALSE))

fitgjrGSEsged = FitMCMC(specgjrGSEsged, data = retGSE, ctr = list())

summary(fitgjrGSEsged)
```

```
########################################
#gjrGARCH(1, 1) with normal distributions KSE
specgjrKSEnorm <- CreateSpec(variance.spec = list(model =
c('gjrGARCH')),distribution.spec = list(distribution = c('norm')),switch.spec =
list(do.mix = FALSE))
fitgjrKSEnorm = FitMCMC(specgjrKSEnorm, data = retKSE, ctr = list())
summary(fitgjrKSEnorm)
#gjrGARCH(1, 1) with skew normal distributions
specgjrKSEsnorm <- CreateSpec(variance.spec = list(model =
c('gjrGARCH')),distribution.spec = list(distribution = c('snorm')),switch.spec =
list(do.mix = FALSE))
fitgjrKSEsnorm = FitMCMC(specgjrKSEsnorm, data = retKSE, ctr = list())
summary(fitgjrKSEsnorm)
#gjrGARCH(1, 1) with std distributions
specgjrKSEstd <- CreateSpec(variance.spec = list(model =
c('gjrGARCH')),distribution.spec = list(distribution = c('std')),switch.spec =
list(do.mix = FALSE))
fitgjrKSEstd = FitMCMC(specgjrKSEstd, data = retKSE, ctr = list())
summary(fitgjrKSEstd)
#gjrGARCH(1, 1) with sstd distributions
specgjrKSEsstd <- CreateSpec(variance.spec = list(model =
c('gjrGARCH')),distribution.spec = list(distribution = c('sstd')),switch.spec =
list(do.mix = FALSE))
fitgjrKSEsstd = FitMCMC(specgjrKSEsstd, data = retKSE, ctr = list())
summary(fitgjrKSEsstd)
```



```
#gjrGARCH(1, 1) with ged distributions
specgjrKSEged <- CreateSpec(variance.spec = list(model =
c('gjrGARCH')),distribution.spec = list(distribution = c('ged')),switch.spec =
list(do.mix = FALSE))
fitgjrKSEged = FitMCMC(specgjrKSEged, data = retKSE, ctr = list())
summary(fitgjrKSEged)
#gjrGARCH(1, 1) with sged distributions
specgjrKSEsged <- CreateSpec(variance.spec = list(model =
c('gjrGARCH')),distribution.spec = list(distribution = c('sged')),switch.spec =
list(do.mix = FALSE))
fitgjrKSEsged = FitMCMC(specgjrKSEsged, data = retKSE, ctr = list())
summary(fitgjrKSEsged)
######################
#gjrGARCH(1, 1) with normal distributions NSE
specgjrNSEnorm <- CreateSpec(variance.spec = list(model =
c('gjrGARCH')),distribution.spec = list(distribution = c('norm')),switch.spec =
list(do.mix = FALSE))
fitgjrNSEnorm = FitMCMC(specgjrNSEnorm, data = retNSE, ctr = list())
summary(fitgjrNSEnorm)
#gjrGARCH(1, 1) with skew normal distributions
specgjrNSEsnorm <- CreateSpec(variance.spec = list(model =
c('gjrGARCH')),distribution.spec = list(distribution = c('snorm')),switch.spec =
list(do.mix = FALSE))
fitgjrNSEsnorm = FitMCMC(specgjrNSEsnorm, data = retNSE, ctr = list())
summary(fitgjrNSEsnorm)
```

```
#gjrGARCH(1, 1) with std distributions
specgjrNSEstd <- CreateSpec(variance.spec = list(model =
c('gjrGARCH')),distribution.spec = list(distribution = c('std')),switch.spec =
list(do.mix = FALSE))
fitgjrNSEstd = FitMCMC(specgjrNSEstd, data = retNSE, ctr = list())
summary(fitgjrNSEstd)
#gjrGARCH(1, 1) with sstd distributions
spec1gjrNSEsstd <- CreateSpec(variance.spec = list(model =
c('gjrGARCH')),distribution.spec = list(distribution = c('sstd')),switch.spec =
list(do.mix = FALSE))
fit1gjrNSEsstd = FitMCMC(spec1gjrNSEsstd, data = retNSE, ctr = list())
summary(fit1gjrNSEsstd)
#gjrGARCH(1, 1) with ged distributions
specgjrNSEged <- CreateSpec(variance.spec = list(model =
c('gjrGARCH')),distribution.spec = list(distribution = c('ged')),switch.spec =
list(do.mix = FALSE))
fitgjrNSEged = FitMCMC(specgjrNSEged, data = retNSE, ctr = list())
summary(fitgjrNSEged)
#gjrGARCH(1, 1) with sged distributions
specgjrNSEsged <- CreateSpec(variance.spec = list(model =
c('gjrGARCH')),distribution.spec = list(distribution = c('sged')),switch.spec =
list(do.mix = FALSE))
fitgjrNSEsged = FitMCMC(specgjrNSEsged, data = retNSE, ctr = list())
summary(fitgjrNSEsged)
###########################################
```

```
#gjrGARCH(1, 1) with normal distributions BSI
specgjrBSInorm <- CreateSpec(variance.spec = list(model =
c('gjrGARCH')),distribution.spec = list(distribution = c('norm')),switch.spec =
list(do.mix = FALSE))
fitgjrBSInorm = FitMCMC(specgjrBSInorm, data = retBSI, ctr = list())
summary(fitgjrBSInorm)
#gjrGARCH(1, 1) with skew normal distributions
specgjrBSIsnorm <- CreateSpec(variance.spec = list(model =
c('gjrGARCH')),distribution.spec = list(distribution = c('snorm')),switch.spec =
list(do.mix = FALSE))
fitgjrBSIsnorm = FitMCMC(specgjrBSIsnorm, data = retBSI, ctr = list())
summary(fitgjrBSIsnorm)
#gjrGARCH(1, 1) with std distributions
specgjrBSIstd <- CreateSpec(variance.spec = list(model =
c('gjrGARCH')),distribution.spec = list(distribution = c('std')),switch.spec =
list(do.mix = FALSE))
fitgjrBSIstd = FitMCMC(specgjrBSIstd, data = retBSI, ctr = list())
summary(fitgjrBSIstd)
#gjrGARCH(1, 1) with sstd distributions
specgjrBSIsstd <- CreateSpec(variance.spec = list(model =
c('gjrGARCH')),distribution.spec = list(distribution = c('sstd')),switch.spec =
list(do.mix = FALSE))
fitgjrBSIsstd = FitMCMC(specgjrBSIsstd, data = retBSI, ctr = list())
summary(fitgjrBSIsstd)
#gjrGARCH(1, 1) with ged distributions
```



```
specgjrBSIged <- CreateSpec(variance.spec = list(model =
c('gjrGARCH')),distribution.spec = list(distribution = c('ged')),switch.spec =
list(do.mix = FALSE))
fitgjrBSIged = FitMCMC(specgjrBSIged, data = retBSI, ctr = list())
summary(fitgjrBSIged)
#gjrGARCH(1, 1) with sged distributions
specgjrBSIsged <- CreateSpec(variance.spec = list(model =
c('gjrGARCH')),distribution.spec = list(distribution = c('sged')),switch.spec =
list(do.mix = FALSE))
fitgjrBSIsged = FitMCMC(specgjrBSIsged, data = retBSI, ctr = list())
summary(fitgjrBSIsged)
########################################
# 2-regime models
# Normal GARCH(1, 1) with normal conditional tails
specsGSEnorm = CreateSpec(variance.spec = list(model = c("sGARCH",
"sGARCH")),distribution.spec = list(distribution = c("norm",
"norm")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitsGSEnorm = FitMCMC(specsGSEnorm, data = retGSE, ctr = list())
summary(fitsGSEnorm)
# Normal GARCH(1, 1) with skewed normal conditional tails
specsGSEsnorm = CreateSpec(variance.spec = list(model = c("sGARCH",
"sGARCH")),distribution.spec = list(distribution = c("snorm",
"snorm")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
```

```
fitsGSEsnorm = FitMCMC(specsGSEsnorm, data = retGSE, ctr = list())
summary(fitsGSEsnorm)
# Normal GARCH(1, 1) with student t normal conditional tails
specsGSEstd = CreateSpec(variance.spec = list(model = c("sGARCH",
"sGARCH")),distribution.spec = list(distribution = c("std", "std")),switch.spec
= list(do.mix = FALSE, K = NULL),constraint.spec = list(fixed = list(),
regime.const = NULL),prior = list(mean = list(), sd = list()))
fitsGSEstd = FitMCMC(specsGSEstd, data = retGSE, ctr = list())
summary(fitsGSEstd)
# Normal GARCH(1, 1) with skewed student t conditional tails
specsGSEsstd = CreateSpec(variance.spec = list(model = c("sGARCH",
"sGARCH")),distribution.spec = list(distribution = c("sstd",
"sstd")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitsGSEsstd = FitMCMC(specsGSEsstd, data = retGSE, ctr = list())
summary(fitsGSEsstd)
# Normal GARCH(1, 1) with ged conditional tails
specsGSEged = CreateSpec(variance.spec = list(model = c("sGARCH",
"sGARCH")),distribution.spec = list(distribution = c("ged",
"ged")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitsGSEged = FitMCMC(specsGSEged, data = retGSE, ctr = list())
summary(fitsGSEged)
# Normal GARCH(1, 1) with skewed ged conditional tails
```



```
specsGSEsged = CreateSpec(variance.spec = list(model = c("sGARCH",
"sGARCH")),distribution.spec = list(distribution = c("sged",
"sged")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitsGSEsged = FitMCMC(specsGSEsged, data = retGSE, ctr = list())
summary(fitsGSEsged)
########################################
# 2-regime models
# E-GARCH(1, 1) with normal conditional tails
speceGSEnorm = CreateSpec(variance.spec = list(model = c("eGARCH",
"eGARCH")),distribution.spec = list(distribution = c("norm",
"norm")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fiteGSEnorm = FitMCMC(speceGSEnorm, data = retGSE, ctr = list())
summary(fiteGSEnorm)
# E-GARCH(1, 1) with skewed normal conditional tails
speceGSEsnorm = CreateSpec(variance.spec = list(model = c("eGARCH",
"eGARCH")),distribution.spec = list(distribution = c("snorm",
"snorm")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fiteGSEsnorm = FitMCMC(speceGSEsnorm, data = retGSE, ctr = list())
summary(fiteGSEsnorm)
# E-GARCH(1, 1) with student t normal conditional tails
speceGSEstd = CreateSpec(variance.spec = list(model = c("eGARCH",
"eGARCH")),distribution.spec = list(distribution = c("std", "std")),switch.spec
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```
= list(do.mix = FALSE, K = NULL),constraint.spec = list(fixed = list(),
regime.const = NULL),prior = list(mean = list(), sd = list()))
fiteGSEstd = FitMCMC(speceGSEstd, data = retGSE, ctr = list())
summary(fiteGSEstd)
# E-GARCH(1, 1) with skewed student t conditional tails
speceGSEsstd = CreateSpec(variance.spec = list(model = c("eGARCH",
"eGARCH")),distribution.spec = list(distribution = c("sstd",
"sstd")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fiteGSEsstd = FitMCMC(speceGSEsstd, data = retGSE, ctr = list())
summary(fiteGSEsstd)
# E-GARCH(1, 1) with ged conditional tails
speceGSEged = CreateSpec(variance.spec = list(model = c("eGARCH",
"eGARCH")),distribution.spec = list(distribution = c("ged",
"ged")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fiteGSEged = FitMCMC(speceGSEged, data = retGSE, ctr = list())
summary(fiteGSEged)
# E-GARCH(1, 1) with skewed ged conditional tails
nmcmc <- 25000
nburn <- 5000
nthin <- 5
ctr <- list(nmcmc = nmcmc, nburn = nburn,nthin = nthin)
speceGSEsged = CreateSpec(variance.spec = list(model = c("eGARCH",
"eGARCH")),distribution.spec = list(distribution = c("sged",
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"sged")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fiteGSEsged = FitMCMC(speceGSEsged, data = retGSE, ctr = ctr)
summary(fiteGSEsged)
########################################
# 2-regime models
# GJR-GARCH(1, 1) with normal conditional tails
specgjrGSEnorm = CreateSpec(variance.spec = list(model = c("gjrGARCH",
"gjrGARCH")),distribution.spec = list(distribution = c("norm",
"norm")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitgjrGSEnorm = FitMCMC(specgjrGSEnorm, data = retGSE, ctr = list())
summary(fitgjrGSEnorm)
# GJR-GARCH(1, 1) with skewed normal conditional tails
specgjrGSEsnorm = CreateSpec(variance.spec = list(model = c("gjrGARCH",
"gjrGARCH")),distribution.spec = list(distribution = c("snorm",
"snorm")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitgjrGSEsnorm = FitMCMC(specgjrGSEsnorm, data = retGSE, ctr = list())
summary(fitgjrGSEsnorm)
# GJR-GARCH(1, 1) with student t normal conditional tails

specgjrGSEstd = CreateSpec(variance.spec = list(model = c("gjrGARCH",
"gjrGARCH")),distribution.spec = list(distribution = c("std",
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"std")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitgjrGSEstd = FitMCMC(specgjrGSEstd, data = retGSE, ctr = list())
summary(fitgjrGSEstd)
# GJR-GARCH(1, 1) with skewed student t conditional tails
specgjrGSEsstd = CreateSpec(variance.spec = list(model = c("gjrGARCH",
"gjrGARCH")),distribution.spec = list(distribution = c("sstd",
"sstd")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitgjrGSEsstd = FitMCMC(specgjrGSEsstd, data = retGSE, ctr = list())
summary(fitgjrGSEsstd)
# GJR-GARCH(1, 1) with ged conditional tails
specgjrGSEged = CreateSpec(variance.spec = list(model = c("gjrGARCH",
"gjrGARCH")),distribution.spec = list(distribution = c("ged",
"ged")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitgjrGSEged = FitMCMC(specgjrGSEged, data = retGSE, ctr = list())
summary(fitgjrGSEged)
# GJR-GARCH(1, 1) with skewed ged conditional tails
specgjrGSEsged = CreateSpec(variance.spec = list(model = c("gjrGARCH",
"gjrGARCH")),distribution.spec = list(distribution = c("sged",
"sged")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitgjrGSEsged = FitMCMC(specgjrGSEsged, data = retGSE, ctr = list(nburn =
5000L, nmcmc = 12500L, nthin = 10L))
```



```
summary(fitgjrGSEsged)

#####################################KSE#####################

# 2-regime models

# Normal GARCH(1, 1) with normal conditional tails

specsKSEnorm = CreateSpec(variance.spec = list(model = c("sGARCH",
"sGARCH")),distribution.spec = list(distribution = c("norm",
"norm")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))

fitsKSEnorm = FitMCMC(specsKSEnorm, data = retKSE, ctr = list())

summary(fitsKSEnorm)

# Normal GARCH(1, 1) with skewed normal conditional tails

specsKSEsnorm = CreateSpec(variance.spec = list(model = c("sGARCH",
"sGARCH")),distribution.spec = list(distribution = c("snorm",
"snorm")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))

fitsKSEsnorm = FitMCMC(specsKSEsnorm, data = retKSE, ctr = list())

summary(fitsKSEsnorm)


# Normal GARCH(1, 1) with student t normal conditional tails

specsKSEstd = CreateSpec(variance.spec = list(model = c("sGARCH",
"sGARCH")),distribution.spec = list(distribution = c("std", "std")),switch.spec
= list(do.mix = FALSE, K = NULL),constraint.spec = list(fixed = list(),
regime.const = NULL),prior = list(mean = list(), sd = list()))

fitsKSEstd = FitMCMC(specsKSEstd, data = retKSE, ctr = list())

summary(fitsKSEstd)
```



```
# Normal GARCH(1, 1) with skewed student t conditional tails
specsKSEsstd = CreateSpec(variance.spec = list(model = c("sGARCH",
"sGARCH")),distribution.spec = list(distribution = c("sstd",
"sstd")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitsKSEsstd = FitMCMC(specsKSEsstd, data = retKSE, ctr = list())
summary(fitsKSEsstd)
#############################Unconditional Volatility of
KSE################
sqrt(250) * sapply(ExtractStateFit(fitsKSEged), UncVol)
# Normal GARCH(1, 1) with ged conditional tails
nmcmc <- 25000
#nmcmc <- 100000
nburn <- 5000
#nthin <- 5
nthin <- 10
ctr <- list(nmcmc = nmcmc, nburn = nburn,nthin = nthin)
specsKSEged = CreateSpec(variance.spec = list(model = c("sGARCH",
"sGARCH")),distribution.spec = list(distribution = c("ged",
"ged")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitsKSEged = FitMCMC(specsKSEged, data = retKSE, ctr = ctr)
summary(fitsKSEged)
plot(fitsKSEged$par[, c(1,2,3,5,6,7)])
# Normal GARCH(1, 1) with skewed ged conditional tails
```

```
specsKSEsged = CreateSpec(variance.spec = list(model = c("sGARCH",
"sGARCH")),distribution.spec = list(distribution = c("sged",
"sged")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitsKSEsged = FitMCMC(specsKSEsged, data = retKSE, ctr = list())
summary(fitsKSEsged)
#########################################
# 2-regime models
# E-GARCH(1, 1) with normal conditional tails
speceKSEnorm = CreateSpec(variance.spec = list(model = c("eGARCH",
"eGARCH")),distribution.spec = list(distribution = c("norm",
"norm")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fiteKSEnorm = FitMCMC(speceKSEnorm, data = retKSE, ctr = list())
summary(fiteKSEnorm)

# E-GARCH(1, 1) with skewed normal conditional tails
speceKSEsnorm = CreateSpec(variance.spec = list(model = c("eGARCH",
"eGARCH")),distribution.spec = list(distribution = c("snorm",
"snorm")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fiteKSEsnorm = FitMCMC(speceKSEsnorm, data = retKSE, ctr = list())
summary(fiteKSEsnorm)
# E-GARCH(1, 1) with student t normal conditional tails
```



```
speceKSEstd = CreateSpec(variance.spec = list(model = c("eGARCH",
"eGARCH")),distribution.spec = list(distribution = c("std", "std")),switch.spec
= list(do.mix = FALSE, K = NULL),constraint.spec = list(fixed = list(),
regime.const = NULL),prior = list(mean = list(), sd = list()))
fiteKSEstd = FitMCMC(speceKSEstd, data = retKSE, ctr = list())
summary(fiteKSEstd)
# E-GARCH(1, 1) with skewed student t conditional tails
speceKSEsstd = CreateSpec(variance.spec = list(model = c("eGARCH",
"eGARCH")),distribution.spec = list(distribution = c("sstd",
"sstd")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fiteKSEsstd = FitMCMC(speceKSEsstd, data = retKSE, ctr = list())
summary(fiteKSEsstd)
# E-GARCH(1, 1) with ged conditional tails
speceKSEged = CreateSpec(variance.spec = list(model = c("eGARCH",
"eGARCH")),distribution.spec = list(distribution = c("ged",
"ged")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fiteKSEged = FitMCMC(speceKSEged, data = retKSE, ctr = list())
summary(fiteKSEged)
# E-GARCH(1, 1) with skewed ged conditional tails
speceKSEsged = CreateSpec(variance.spec = list(model = c("eGARCH",
"eGARCH")),distribution.spec = list(distribution = c("sged",
"sged")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
```

```
fiteKSEsged = FitMCMC(speceKSEsged, data = retKSE, ctr = list())
summary(fiteKSEsged)
########################################
# 2-regime models
# GJR-GARCH(1, 1) with normal conditional tails
specgjrKSEnorm = CreateSpec(variance.spec = list(model = c("gjrGARCH",
"gjrGARCH")),distribution.spec = list(distribution = c("norm",
"norm")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitgjrKSEnorm = FitMCMC(specgjrKSEnorm, data = retKSE, ctr = list())
summary(fitgjrKSEnorm)
# GJR-GARCH(1, 1) with skewed normal conditional tails
specgjrKSEsnorm = CreateSpec(variance.spec = list(model = c("gjrGARCH",
"gjrGARCH")),distribution.spec = list(distribution = c("snorm",
"snorm")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitgjrKSEsnorm = FitMCMC(specgjrKSEsnorm, data = retKSE, ctr = list())
summary(fitgjrKSEsnorm)
# GJR-GARCH(1, 1) with student t normal conditional tails
specgjrKSEstd = CreateSpec(variance.spec = list(model = c("gjrGARCH",
"gjrGARCH")),distribution.spec = list(distribution = c("std",
"std")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitgjrKSEstd = FitMCMC(specgjrKSEstd, data = retKSE, ctr = list())
summary(fitgjrKSEstd)
```

```
# GJR-GARCH(1, 1) with skewed student t conditional tails
specgjrKSEsstd = CreateSpec(variance.spec = list(model = c("gjrGARCH",
"gjrGARCH")),distribution.spec = list(distribution = c("sstd",
"sstd")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitgjrKSEsstd = FitMCMC(specgjrKSEsstd, data = retKSE, ctr = list())
summary(fitgjrKSEsstd)
# GJR-GARCH(1, 1) with ged conditional tails
specgjrKSEged = CreateSpec(variance.spec = list(model = c("gjrGARCH",
"gjrGARCH")),distribution.spec = list(distribution = c("ged",
"ged")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitgjrKSEged = FitMCMC(specgjrKSEged, data = retKSE, ctr = list())
summary(fitgjrKSEged)


# GJR-GARCH(1, 1) with skewed ged conditional tails
specgjrKSEsged = CreateSpec(variance.spec = list(model = c("gjrGARCH",
"gjrGARCH")),distribution.spec = list(distribution = c("sged",
"sged")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitgjrKSEsged = FitMCMC(specgjrKSEsged, data = retKSE, ctr = list())
summary(fitgjrKSEsged)
########################################
# 2-regime models for NSE
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```
# Normal GARCH(1, 1) with normal conditional tails
specsNSEnorm = CreateSpec(variance.spec = list(model = c("sGARCH",
"sGARCH")),distribution.spec = list(distribution = c("norm",
"norm")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitsNSEnorm = FitMCMC(specsNSEnorm, data = retNSE, ctr = list())
summary(fitsNSEnorm)
# Normal GARCH(1, 1) with skewed normal conditional tails
specsNSEsnorm = CreateSpec(variance.spec = list(model = c("sGARCH",
"sGARCH")),distribution.spec = list(distribution = c("snorm",
"snorm")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitsNSEsnorm = FitMCMC(specsNSEsnorm, data = retNSE, ctr = list())
summary(fitsNSEsnorm)
# Normal GARCH(1, 1) with student t normal conditional tails


specsNSEstd = CreateSpec(variance.spec = list(model = c("sGARCH",
"sGARCH")),distribution.spec = list(distribution = c("std", "std")),switch.spec
= list(do.mix = FALSE, K = NULL),constraint.spec = list(fixed = list(),
regime.const = NULL),prior = list(mean = list(), sd = list()))
fitsNSEstd = FitMCMC(specsNSEstd, data = retNSE, ctr = list())
summary(fitsNSEstd)
# Normal GARCH(1, 1) with skewed student t conditional tails
specsNSEsstd = CreateSpec(variance.spec = list(model = c("sGARCH",
"sGARCH")),distribution.spec = list(distribution = c("sstd",
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"sstd")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitsNSEsstd = FitMCMC(specsNSEsstd, data = retNSE, ctr = list())
summary(fitsNSEsstd)
# Normal GARCH(1, 1) with ged conditional tails
specsNSEged = CreateSpec(variance.spec = list(model = c("sGARCH",
"sGARCH")),distribution.spec = list(distribution = c("ged",
"ged")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitsNSEged = FitMCMC(specsNSEged, data = retNSE, ctr = list())
summary(fitsNSEged)
# Normal GARCH(1, 1) with skewed ged conditional tails
specsNSEsged = CreateSpec(variance.spec = list(model = c("sGARCH",
"sGARCH")),distribution.spec = list(distribution = c("sged",
"sged")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitsNSEsged = FitMCMC(specsNSEsged, data = retNSE, ctr = list())
summary(fitsNSEsged)
########################################
# 2-regime models
# E-GARCH(1, 1) with normal conditional tails
speceNSEnorm = CreateSpec(variance.spec = list(---model = c("eGARCH",
"eGARCH")),distribution.spec = list(distribution = c("norm",
"norm")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
```

```
fiteNSEnorm = FitMCMC(speceNSEnorm, data = retNSE, ctr = list())
summary(fiteNSEnorm)
# E-GARCH(1, 1) with skewed normal conditional tails
speceNSEsnorm = CreateSpec(variance.spec = list(model = c("eGARCH",
"eGARCH")),distribution.spec = list(distribution = c("snorm",
"snorm")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fiteNSEsnorm = FitMCMC(speceNSEsnorm, data = retNSE, ctr = list())
summary(fiteNSEsnorm)
# E-GARCH(1, 1) with student t normal conditional tails
speceNSEstd = CreateSpec(variance.spec = list(model = c("eGARCH",
"eGARCH")),distribution.spec = list(distribution = c("std", "std")),switch.spec
= list(do.mix = FALSE, K = NULL),constraint.spec = list(fixed = list(),
regime.const = NULL),prior = list(mean = list(), sd = list()))
fiteNSEstd = FitMCMC(speceNSEstd, data = retNSE, ctr = list())
summary(fiteNSEstd)
# E-GARCH(1, 1) with skewed student t conditional tails
speceNSEsstd = CreateSpec(variance.spec = list(model = c("eGARCH",
"eGARCH")),distribution.spec = list(distribution = c("sstd",
"sstd")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fiteNSEsstd = FitMCMC(speceNSEsstd, data = retNSE, ctr = list())
summary(fiteNSEsstd)
# E-GARCH(1, 1) with ged conditional tails
```

```
speceNSEged = CreateSpec(variance.spec = list(model = c("eGARCH",
"eGARCH")),distribution.spec = list(distribution = c("ged",
"ged")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fiteNSEged = FitMCMC(speceNSEged, data = retNSE, ctr = list())
summary(fiteNSEged)
# E-GARCH(1, 1) with skewed ged conditional tails
speceNSEsged = CreateSpec(variance.spec = list(model = c("eGARCH",
"eGARCH")),distribution.spec = list(distribution = c("sged",
"sged")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fiteNSEsged = FitMCMC(speceNSEsged, data = retNSE, ctr = list())
summary(fiteNSEsged)
########################################
# 2-regime models
# GJR-GARCH(1, 1) with normal conditional tails

specgjrNSEnorm = CreateSpec(variance.spec = list(model = c("gjrGARCH",
"gjrGARCH")),distribution.spec = list(distribution = c("norm",
"norm")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitgjrNSEnorm = FitMCMC(specgjrNSEnorm, data = returnsIndices[,'NSE'],
ctr = list())
summary(fitgjrNSEnorm)
# GJR-GARCH(1, 1) with skewed normal conditional tails
```

```
specgjrNSEsnorm = CreateSpec(variance.spec = list(model = c("gjrGARCH",
"gjrGARCH")),distribution.spec = list(distribution = c("snorm",
"snorm")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitgjrNSEsnorm = FitMCMC(specgjrNSEsnorm, data = returnsIndices[,'NSE'],
ctr = list())
summary(fitgjrNSEsnorm)
# GJR-GARCH(1, 1) with student t normal conditional tails
specgjrNSEstd = CreateSpec(variance.spec = list(model = c("gjrGARCH",
"gjrGARCH")),distribution.spec = list(distribution = c("std",
"std")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitgjrNSEstd = FitMCMC(specgjrNSEstd, data = returnsIndices[,'NSE'], ctr =
list())
summary(fitgjrNSEstd)
# GJR-GARCH(1, 1) with skewed student t conditional tails

specgjrNSEsstd = CreateSpec(variance.spec = list(model = c("gjrGARCH",
"gjrGARCH")),distribution.spec = list(distribution = c("sstd",
"sstd")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitgjrNSEsstd = FitMCMC(specgjrNSEsstd, data = returnsIndices[,'NSE'], ctr
= list(nburn = 5000L, nmcmc = 15000L, nthin = 10L))
summary(fitgjrNSEsstd)
# GJR-GARCH(1, 1) with ged conditional tails
```



```
specgjrNSEged = CreateSpec(variance.spec = list(model = c("gjrGARCH",
"gjrGARCH")),distribution.spec = list(distribution = c("ged",
"ged")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitgjrNSEged = FitMCMC(specgjrNSEged, data = returnsIndices[,'NSE'], ctr =
list())
summary(fitgjrNSEged)
# GJR-GARCH(1, 1) with skewed ged conditional tails
specgjrNSEsged = CreateSpec(variance.spec = list(model = c("gjrGARCH",
"gjrGARCH")),distribution.spec = list(distribution = c("sged",
"sged")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitgjrNSEsged = FitMCMC(specgjrNSEsged, data = returnsIndices[,'NSE'], ctr
= list())
summary(fitgjrNSEsged)
##########################################
# 2-regime models
# Normal GARCH(1, 1) with normal conditional tails
ret = returnsIndices[,'BSI'] #- mean(returnsIndices[,'BSI'])
retBSI = ret * 100
specsBSInorm = CreateSpec(variance.spec = list(model = c("sGARCH",
"sGARCH")),distribution.spec = list(distribution = c("norm",
"norm")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitsBSInorm = FitMCMC(specsBSInorm, data = retBSI, ctr = list())
```



```
summary(fitsBSInorm)
# Normal GARCH(1, 1) with skewed normal conditional tails
specsBSIsnorm = CreateSpec(variance.spec = list(model = c("sGARCH",
"sGARCH")),distribution.spec = list(distribution = c("snorm",
"snorm")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitsBSIsnorm = FitMCMC(specsBSIsnorm, data = retBSI, ctr = list())
summary(fitsBSIsnorm)
# Normal GARCH(1, 1) with student t normal conditional tails
specsBSIstd = CreateSpec(variance.spec = list(model = c("sGARCH",
"sGARCH")),distribution.spec = list(distribution = c("std", "std")),switch.spec
= list(do.mix = FALSE, K = NULL),constraint.spec = list(fixed = list(),
regime.const = NULL),prior = list(mean = list(), sd = list()))
fitsBSIstd = FitMCMC(specsBSIstd, data = retBSI, ctr = list())
summary(fitsBSIstd)


# Normal GARCH(1, 1) with skewed student t conditional tails
specsBSIsstd = CreateSpec(variance.spec = list(model = c("sGARCH",
"sGARCH")),distribution.spec = list(distribution = c("sstd",
"sstd")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitsBSIsstd = FitMCMC(specsBSIsstd, data = retBSI, ctr = list())
summary(fitsBSIsstd)
# Normal GARCH(1, 1) with ged conditional tails
```

```
specsBSIged = CreateSpec(variance.spec = list(model = c("sGARCH",
"sGARCH")),distribution.spec = list(distribution = c("ged",
"ged")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitsBSIged = FitMCMC(specsBSIged, data = retBSI, ctr = list())
summary(fitsBSIged)
# Normal GARCH(1, 1) with skewed ged conditional tails
specsBSIsged = CreateSpec(variance.spec = list(model = c("sGARCH",
"sGARCH")),distribution.spec = list(distribution = c("sged",
"sged")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitsBSIsged = FitMCMC(specsBSIsged, data = returnsIndices[,'BSI'], ctr =
list())
summary(fitsBSIsged)
########################################
# 2-regime models
# E-GARCH(1, 1) with normal conditional tails
speceBSInorm = CreateSpec(variance.spec = list(model = c("eGARCH",
"eGARCH")),distribution.spec = list(distribution = c("norm",
"norm")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fiteBSInorm = FitMCMC(speceBSInorm, data = returnsIndices[,'BSI'], ctr =
list())
summary(fiteBSInorm)
# E-GARCH(1, 1) with skewed normal conditional tails
```

speceBSIsnorm = CreateSpec(variance.spec = list(model = c("eGARCH", "eGARCH")),distribution.spec = list(distribution = c("snorm", "snorm")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec = list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))

fiteBSIsnorm = FitMCMC(speceBSIsnorm, data = returnsIndices[,'BSI'], ctr = list())

summary(fiteBSIsnorm)

E-GARCH(1, 1) with student t normal conditional tails

speceBSIstd = CreateSpec(variance.spec = list(model = c("eGARCH", "eGARCH")),distribution.spec = list(distribution = c("std", "std")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec = list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))

fiteBSIstd = FitMCMC(speceBSIstd, data = returnsIndices[,'BSI'], ctr = list())

summary(fiteBSIstd)

E-GARCH(1, 1) with skewed student t conditional tails

speceBSIsstd = CreateSpec(variance.spec = list(model = c("eGARCH", "eGARCH")),distribution.spec = list(distribution = c("sstd", "sstd")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec = list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))

fiteBSIsstd = FitMCMC(speceBSIsstd, data = returnsIndices[,'BSI'], ctr = list())

summary(fiteBSIsstd)

E-GARCH(1, 1) with ged conditional tails


```
speceBSIged = CreateSpec(variance.spec = list(model = c("eGARCH",
"eGARCH")),distribution.spec = list(distribution = c("ged",
"ged")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fiteBSIged = FitMCMC(speceBSIged, data = returnsIndices[,'BSI'], ctr =
list())
summary(fiteBSIged)
# E-GARCH(1, 1) with skewed ged conditional tails
speceBSIsged = CreateSpec(variance.spec = list(model = c("eGARCH",
"eGARCH")),distribution.spec = list(distribution = c("sged",
"sged")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fiteBSIsged = FitMCMC(speceBSIsged, data = returnsIndices[,'BSI'], ctr =
list())
summary(fiteBSIsged)
########################################
# 2-regime models
# GJR-GARCH(1, 1) with normal conditional tails
specgjrBSInorm = CreateSpec(variance.spec = list(model = c("gjrGARCH",
"gjrGARCH")),distribution.spec = list(distribution = c("norm",
"norm")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitgjrBSInorm = FitMCMC(specgjrBSInorm, data = returnsIndices[,'BSI'], ctr
= list())
summary(fitgjrBSInorm)
```

```
# GJR-GARCH(1, 1) with ged conditional tails
specgjrBSIged = CreateSpec(variance.spec = list(model = c("gjrGARCH",
"gjrGARCH")),distribution.spec = list(distribution = c("ged",
"ged")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitgjrBSIged = FitMCMC(specgjrBSIged, data = returnsIndices[,'BSI'], ctr =
list())
summary(fitgjrBSIged)
# GJR-GARCH(1, 1) with skewed ged conditional tails
specgjrBSIsged = CreateSpec(variance.spec = list(model = c("gjrGARCH",
"gjrGARCH")),distribution.spec = list(distribution = c("sged",
"sged")),switch.spec = list(do.mix = FALSE, K = NULL),constraint.spec =
list(fixed = list(), regime.const = NULL),prior = list(mean = list(), sd = list()))
fitgjrBSIsged = FitMCMC(specgjrBSIsged, data = returnsIndices[,'BSI'], ctr =
list())
summary(fitgjrBSIsged)

# Traceplots for convergence
library(coda)
fiteGSEsged$par[, 1]
traceplot(fiteGSEsged$par[, 2])
traceplot(fiteGSEsged$par, smooth = FALSE, col = 1, type = 'l', xlab =
'Iterations', ylab = '')
traceplot(fiteGSEsged$par[, 1:10], smooth = FALSE, type = 'l', xlab =
'Iterations', ylab = '')
```

```
par(mfrow = c(2,4))
traceplot(fiteGSEsged$par[,1], smooth = TRUE)
quantile(fiteGSEsged$par[, 1], probs = c(0.025, 0.975))
summary(fiteGSEsged)
names(fiteGSEsged)
plot(fiteGSEsged$par[,c(1:4, 7:10)])
plot(fitsKSEged$par[, c(1:3, 5:7)])
plot(fitgjrNSEsstd$par[, c(1:4, 7:10)])
traceplot(fitgjrNSEsstd$par[, c(1:4, 7:10)])
library(MSGARCH)
################################ Backtesting GSE
#Create the single regime specification
spec1SGSEsged <- CreateSpec(variance.spec = list(model = c('sGARCH')),
distribution.spec = list(distribution = c('sged')), switch.spec = list(K = 1))
models = list(spec1SGSEsged, speceGSEsged)
n.ots <- 548
n.its <- 1000
alpha <- 0.05
k.update <- 10
#1548 length(retGSE)
VaR <- matrix(NA, nrow = n.ots, ncol = length(models))
y.ots <- matrix(NA, nrow = n.ots, ncol = 1)
model.fit <- vector(mode = 'list', length = length(models))
for (i in 1:n.ots) {
cat("Backtest - Iteration: ", i, "\n")
```

```
y.its <- retGSE[i:(n.its + i - 1)]
y.ots[i] <- retGSE[n.its + i]
for (j in 1:length(models)) {
 if (k.update == 1 || i %% k.update == 1) {
 cat("Model", j, "is reestimated\n")
 model.fit[[j]] <- FitML(spec = models[[j]], data = y.its,
 ctr = list(do.se = FALSE))
 }
VaR[i,j] <- Risk(model.fit[[j]]$spec, par = model.fit[[j]]$par,data = y.its,
n.ahead = 1, alpha = alpha,do.es = FALSE, do.its = FALSE)$VaR
}
}
library("zoo")
time.index <- zoo::index(retGSE)[(n.its + 1):(n.ots + n.its)]
y_ots <- zoo::zoo(y.ots, order.by = time.index)
VaR   <- zoo::zoo(VaR, order.by = time.index)


par(mfrow = c(2, 2))
plot(y_ots, type = 'p', las = 0.5, lwd = 0.5, xlab = 'Date',ylab = '', col = 'black',
cex.axis = 1.5, cex.lab = 1.5, pch = 1.5)
lines(VaR[,1], type = 'l', col = 'red', lwd = 0.8, lty = 'dashed')
lines(VaR[,2], type = 'l', col = 'blue', lwd = 0.8)
legend('topleft', legend = c('VaR 5% - EGARCG-sged', 'VaR 5% - MS2-
EGARCH-sged'), col = c('red', 'blue'), lwd = 2, cex = 1.0, lty = c('dashed',
'solid'))
```

```
abline(h = 0, lwd = 1)
title('Backtesing GSE Returns VaR at 5% risk level', cex.main = 1.0)
library(GAS)
CC.pval <- DQ.pval <- vector('double', length(models))
for (j in 1:length(models)) {
 test <- GAS::BacktestVaR(data = y.ots, VaR = VaR[,j], alpha = alpha)
 CC.pval[j] <- test$LRcc[2]
 DQ.pval[j] <- test$DQ$pvalue
 }
names(CC.pval) <- names(DQ.pval) <- c('GARCH-sged', 'MS2-EGARCH-
sged')
print(CC.pval)
print(DQ.pval)
################################Unconditional Volatility of GSE
sqrt(250) * sapply(ExtractStateFit(fiteGSEsged), UncVol)
plot(fiteGSEsged$par[, c(1, 2, 3, 4, 7, 8, 9, 10)])
vol <- sqrt(250) * Volatility(fiteGSEsged)
plot(vol)
################################# Backtesting KSE
#Create the single regime specification
spec1KSEsstd <- CreateSpec(variance.spec = list(model = c('sGARCH')),
distribution.spec = list(distribution = c('sstd')), switch.spec = list(K = 1))
models = list(spec1KSEsstd, specsKSEged)
n.ots <- 548
n.its <- 1000
```

```
alpha <- 0.05
k.update <- 10
#1548 length(retKSE)
VaR <- matrix(NA, nrow = n.ots, ncol = length(models))
y.ots <- matrix(NA, nrow = n.ots, ncol = 1)
model.fit <- vector(mode = 'list', length = length(models))
for (i in 1:n.ots) {
cat("Backtest - Iteration: ", i, "\n")
y.its <- retKSE[i:(n.its + i - 1)]
y.ots[i] <- retKSE[n.its + i]
for (j in 1:length(models)) {
if (k.update == 1 || i %% k.update == 1) {
cat("Model", j, "is reestimated\n")
model.fit[[j]] <- FitML(spec = models[[j]], data = y.its,
ctr = list(do.se = FALSE))
}
VaR[i,j] <- Risk(model.fit[[j]]$spec, par = model.fit[[j]]$par,data = y.its,
n.ahead = 1, alpha = alpha,do.es = FALSE, do.its = FALSE)$VaR
}
}
# Fig
library("zoo")
time.index <- zoo::index(retKSE)[(n.its + 1):(n.ots + n.its)]
y_ots <- zoo::zoo(y.ots, order.by = time.index)
VaR  <- zoo::zoo(VaR, order.by = time.index)
```

```
par(mfrow = c(2, 2))
plot(y_ots, type = 'p', las = 0.5, lwd = 0.5, xlab = 'Date',ylab = '', col = 'black',
cex.axis = 1.5, cex.lab = 1.5, pch = 1.5)
lines(VaR[,1], type = 'l', col = 'red', lwd = 0.8, lty = 'dashed')
lines(VaR[,2], type = 'l', col = 'blue', lwd = 0.8)
legend('topleft', legend = c('VaR 5% - GARCH-sstd', 'VaR 5% - MS2-
GARCH-ged'), col = c('red', 'blue'), lwd = 2, cex = 1.0, lty = c('dashed',
'solid'))
abline(h = 0, lwd = 1)
title('Backtesing KSE Returns VaR at 5% risk level', cex.main = 1.0)
library(GAS)
CC.pval <- DQ.pval <- vector('double', length(models))
for (j in 1:length(models)) {
 test <- GAS::BacktestVaR(data = y.ots, VaR = VaR[,j], alpha = alpha)
 CC.pval[j] <- test$LRcc[2]
 DQ.pval[j] <- test$DQ$pvalue
 }
names(CC.pval) <- names(DQ.pval) <- c('EGARCH-sged', 'MS2-EGARCH-
sged')
print(CC.pval)
print(DQ.pval)
################################ Backtesting NSE
#Create the single regime specification
spec1gjrNSEsstd <- CreateSpec(variance.spec = list(model = c('gjrGARCH')),
distribution.spec = list(distribution = c('sstd')), switch.spec = list(K = 1))
```

```
models = list(spec1gjrNSEsstd, specgjrNSEsstd)
n.ots <- 548
n.its <- 1000
alpha <- 0.05
k.update <- 10
#1548 length(retNSE)
VaR <- matrix(NA, nrow = n.ots, ncol = length(models))
y.ots <- matrix(NA, nrow = n.ots, ncol = 1)
model.fit <- vector(mode = 'list', length = length(models))
for (i in 1:n.ots) {
cat("Backtest - Iteration: ", i, "\n")
y.its <- retNSE[i:(n.its + i - 1)]
y.ots[i] <- retNSE[n.its + i]
for (j in 1:length(models)) {
if (k.update == 1 || i %% k.update == 1) {
cat("Model", j, "is reestimated\n")
model.fit[[j]] <- FitML(spec = models[[j]], data = y.its,
ctr = list(do.se = FALSE))
}
VaR[i,j] <- Risk(model.fit[[j]]$spec, par = model.fit[[j]]$par,data = y.its,
n.ahead = 1, alpha = alpha,do.es = FALSE, do.its = FALSE)$VaR
}
}
library("zoo")
time.index <- zoo::index(retNSE)[(n.its + 1):(n.ots + n.its)]
```

```
y_ots <- zoo::zoo(y.ots, order.by = time.index)
VaR   <- zoo::zoo(VaR, order.by = time.index)
#par(mfrow = c(2, 2))
plot(y_ots, type = 'p', las = 0.5, lwd = 0.5, xlab = 'Date',ylab = '', col = 'black',
cex.axis = 1.5, cex.lab = 1.5, pch = 1.5)
lines(VaR[,1], type = 'l', col = 'red', lwd = 0.8, lty = 'dashed')
lines(VaR[,2], type = 'l', col = 'blue', lwd = 0.8)
legend('bottomleft', legend = c('VaR 5% - GJR-GARCH-sstd', 'VaR 5% -
MS2-GJR-GARCH-sstd'), col = c('red', 'blue'), lwd = 2, cex = 1.0, lty =
c('dashed', 'solid'))
abline(h = 0, lwd = 1)
title('Backtesing NSE Returns VaR at 5% risk level', cex.main = 1.0)
library(GAS)
CC.pval <- DQ.pval <- vector('double', length(models))
for (j in 1:length(models)) {
 test <- GAS::BacktestVaR(data = y.ots, VaR = VaR[,j], alpha = alpha)
 CC.pval[j] <- test$LRcc[2]
 DQ.pval[j] <- test$DQ$pvalue
 }
names(CC.pval) <- names(DQ.pval) <- c('GJR-GARCH-sstd', 'MS2-GJR-
GARCH-sstd')
print(CC.pval)
print(DQ.pval)
################################# Backtesting BSI
#Create the single regime specification
```

```
spec1BSIsged <- CreateSpec(variance.spec = list(model = c('sGARCH')),
distribution.spec = list(distribution = c('sged')), switch.spec = list(K = 1))
models = list(spec1BSIsged, specsBSIsstd)
n.ots <- 548
n.its <- 1000
alpha <- 0.05
k.update <- 10
#1548 length(retBSI)
VaR <- matrix(NA, nrow = n.ots, ncol = length(models))
y.ots <- matrix(NA, nrow = n.ots, ncol = 1)
model.fit <- vector(mode = 'list', length = length(models))
for (i in 1:n.ots) {
cat("Backtest - Iteration: ", i, "\n")
y.its <- retBSI[i:(n.its + i - 1)]
y.ots[i] <- retBSI[n.its + i]
for (j in 1:length(models)) {
if (k.update == 1 || i %% k.update == 1) {
cat("Model", j, "is reestimated\n")
model.fit[[j]] <- FitML(spec = models[[j]], data = y.its,
ctr = list(do.se = FALSE))
}
VaR[i,j] <- Risk(model.fit[[j]]$spec, par = model.fit[[j]]$par,data = y.its,
n.ahead = 1, alpha = alpha,do.es = FALSE, do.its = FALSE)$VaR
}
}
```

```
library("zoo")

time.index <- zoo::index(retBSI)[(n.its + 1):(n.ots + n.its)]

y_ots <- zoo::zoo(y.ots, order.by = time.index)

VaR  <- zoo::zoo(VaR, order.by = time.index)

plot(y_ots, type = 'p', las = 0.5, lwd = 0.5, xlab = 'Date',ylab = '', col = 'black',
cex.axis = 1.5, cex.lab = 1.5, pch = 1.5)

lines(VaR[,1], type = 'l', col = 'red', lwd = 0.8, lty = 'dashed')

lines(VaR[,2], type = 'l', col = 'blue', lwd = 0.8)

legend('topright', legend = c('VaR 5% - GARCH-sstd', 'VaR 5% - MS2-
GARCH-sstd'), col = c('red', 'blue'), lwd = 2, cex = 1.0, lty = c('dashed',
'solid'))

abline(h = 0, lwd = 1)

title('Backtesing BSI Returns VaR at 5% risk level', cex.main = 1.0)

library(GAS)

CC.pval <- DQ.pval <- vector('double', length(models))

for (j in 1:length(models)) {

 test <- GAS::BacktestVaR(data = y.ots, VaR = VaR[,j], alpha = alpha)

 CC.pval[j] <- test$LRcc[2]

}

names(CC.pval) <- names(DQ.pval) <- c('GARCH-sged', 'MS2-GARCH-
sstd')

print(CC.pval)

print(DQ.pval)
```

APPENDIX C: PAPERS

1. **Korkpoe, C. H., & Howard, N. (2019).** Volatility Model Choice for Sub-Saharan Frontier Equity Markets-A Markov Regime Switching Bayesian Approach. *EMAJ: Emerging Markets Journal*, *9*(1), 69-79.

Abstract

We adopt a granular approach to estimating the risk of equity returns in sub-Saharan African frontier equity markets under the assumption that, returns are influenced by developments in the underlying economy. Four countries were studied – Botswana, Ghana, Kenya and Nigeria. We found heterogeneity in the evolution of volatility across these markets and also that two-regime switching volatility models describe better the heteroscedastic returns generating processes in these markets using the deviance information criteria. We backtest the results to assess whether the models are a good fit for the data. We concluded that, the selected models are the most suitable for predicting the volatility of future returns in the markets studied.

2. **Korkpoe, C. H., & Howard, N. (2019).** Regime Detection in Sub-Saharan Africa Equity Markets – A Hidden Markov Model Approach. *Journal of African Business (Under review)*

Complaints of heightened risks in the sub-Saharan African equities markets are rife in the practitioner literature. Investors need an understanding of the volatility dynamics in these frontier markets. This paper uses the Hidden Markov Models to detect the points of regime changes in the volatility in the markets of Ghana, Kenya, Nigerian and Botswana. We used the daily closing index of the exchanges and modeled 2- and 3-regimes in the market. Information criteria selected the best fitting model of 2-regime changes

corresponding to periods of low and high volatilities. This has been shown through smoothed volatility plots depicting times of regime changes over the sample periods. Investors will be guided in the strategies they choose by setting prices filters according to the particular regimes. For regulators, the work will help in setting risk sensitive capital based on market regimes so that firms do not carry too much capital than it is required.